## Electroweak chiral Lagrangian for $W^{\prime}$ boson

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Abstract: The complete list of electroweak chiral Lagrangian for $W^{\prime}, Z^{\prime}$ and a neutral light higgs with symmetry $\mathrm{SU}(2)_{1} \otimes \mathrm{SU}(2)_{2} \otimes \mathrm{U}(1)$ is provided. The bosonic part is accurate up to order of $p^{4}$, the matter part involving various fermion representation arrangements includes dimension three Yukawa type and dimension four gauge type operators. The universal mixings and masses of gauge bosons and fermions are given. Constraints from mass differences for $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}, B_{s}^{0}-\bar{B}_{s}^{0}$ systems and indirect CP violation parameter $\left|\epsilon_{K}\right|$ for $K$ mesons are evaluated.

Keywords: Beyond Standard Model, Chiral Lagrangians, Gauge Symmetry, Kaon Physics.

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## 1. Introduction

Any electrically charged gauge boson outside of the Standard Model (SM) is generically denoted $W^{\prime}$. It is a hypothetical massive particle of electric charge $\pm 1$ and spin 1 which always couples to two different flavors of quarks and (or) leptons, similar to the $W$ boson (We do not discuss the situation that $W^{\prime}$ as a leptoquark gauge boson couples quarks to leptons). $W^{\prime}$ can be seen as minimal charged gauge boson extension for SM and is predicted in various new physics models, such as Left-Right symmetric models [1, 21, Alternate Left-Right model [3], Ununified standard model [4], Non-Commuting Extended Technicolor [5], Little Higgs models [6-8], Higgsless models [9], models of composite gauge bosons [10], Supersymmetric top-flavor models [11, Grand Unification 12] and Superstring theories 13-15, Extra-dimensions [16, 17]. Theoretically, unitarity considerations imply that charged massive vector $W^{\prime}$ s are gauge bosons associated with some spontaneously broken non-abelian gauge symmetry [18]. This is true even when it is a composite particle like the charged techni- $\rho$ in technicolor theories [19] or a Kaluza-Klein mode in theories where the $W$ boson propagates in extra dimensions 20]. The minimal rank one non-abelian gauge group is $\mathrm{SU}(2)$. Besides $W^{\prime \pm}$, the group $\mathrm{SU}(2)$ demands the existence of extra neutral gauge boson $Z^{\prime}$. $W^{\prime \pm}$ and $Z^{\prime}$ together form a consistent minimal non-abelian $\mathrm{SU}(2)$ gauge group. This gauge group must be completely spontaneously broken to give $W^{\prime \pm}, Z^{\prime}$ masses through Higgs mechanism. The breaking mechanism is not known yet which depends on detail of
the model. We can exploit nonlinear realization of the symmetry to avoid touching upon the details of the breaking mechanism. This is the $\mathrm{SU}(2)$ chiral Lagrangian for $W^{\prime \pm}, Z^{\prime}$ and three corresponding Goldstone bosons.

Now the new generation hadron collider LHC is going to run and people are eager expecting the discovery of the new particles. Once the first new particle shows its signature in the collider experiment and its spin and parity are evaluated out, the following work is to check whether it belongs to any of exiting models. In general, for each kind of possible new particle, there are many candidate models predicting it and waiting for experiment to check. It is also possible that the real model our nature chosen is not presented in this candidate's list. To examine which kind of model this new particle belongs to and its interactions with those already discovered particles, we need a phenomenological theory which must be such general as to include various underlying discovered and undiscovered candidate models and cover all of its possible phenomenologies. We call this phenomenological theory the electroweak chiral Lagrangian (EWCL) for the new particle which include this new particle and all those already discovered particles. The symmetry realization of this EWCL should at least include $\mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y}$ plus some new part from the new particle. On the platform of this EWCL, on the one hand, we can perform model independent phenomenological investigation of the new particle and fix the corresponding parameters in EWCL from experiments, on the other hand, we can compute the parameters of EWCL from concrete underlying models. Through comparison between parameters from experiments and that from underlying model, we hope the correct underlying model can be figured out.

In this paper, we are interested in a situation that except discovered particles in SM, the lowest new particles which are expected to show up in upcoming collider experiments are $W^{\prime \pm}$ and $Z^{\prime}$. According to discussions above, to describe the corresponding physics phenomenologically, we are lead to set up a EWCL for $W^{\prime \pm}, Z^{\prime}$ and the symmetry realization of the theory will be generalized from original $\mathrm{SU}(2)_{L} \otimes \mathrm{U}(1)_{Y}$ to $\mathrm{SU}(2)_{1} \otimes \mathrm{SU}(2)_{2} \otimes \mathrm{U}(1)$ for which one $\mathrm{SU}(2)$ is for $W^{\prime \pm}$ and $Z^{\prime}$ and remaining ones are for SM electro-weak gauge bosons $W^{ \pm}, Z, A$. Naive extension of conventional unitarity analysis shows that this Lagrangian will violate unitarity in TeV energy region [21, and adding in theory a neutral Higgs with mass below TeV will kill the disaster. To keep our theory being unitary at TeV energy region, we will further include in our theory a neutral Higgs. Thus our EWCL for $W^{\prime \pm}, Z^{\prime}$ now will include those already discovered particles, a neutral Higgs, $W^{\prime \pm}, Z^{\prime}$ and corresponding Goldstone bosons. In fact, without $W^{\prime \pm}, Z^{\prime}$ and corresponding Goldstone bosons, the EWCL only for a neutral Higgs boson was already written down in ref. [22] which was a generalization of original standard EWCL [23-25] by adding a singlet Higgs field to the theory. Now our EWCL can be seen as a further extension of this generalized EWCL to include in theory $W^{\prime \pm}, Z^{\prime}$ and corresponding Goldstone bosons. In this work, we are especially interested in the case that the mass of $W^{\prime \pm}$ is lighter or roughly same as that of $Z^{\prime}$. Since if the mass of $Z^{\prime}$ is much lighter than that of $W^{\prime \pm}$, the phenomenological interest will be changed to physics for lighter $Z^{\prime}$. The heavier $W^{\prime \pm}$ then can be integrated out theoretically and we are led to EWCL purely for $Z^{\prime}$ and neutral Higgs boson. This EWCL was already discussed by us in another paper [26] in which $Z^{\prime}$ can be either an element of $\operatorname{SU}(2)$ triplet or a remnant of some other underlying dynamics which
has nothing to do with $W^{\prime}$ and can not be covered in our present theory. It is shown in ref. [26] that EWCL for $Z^{\prime}$ is equivalent to an extended Stueckelberg mechanism for $\mathrm{U}(1)$ gauge boson. From the point of view of Stueckelberg mechanism, our present EWCL for $W^{\prime}$ and $Z^{\prime}$ can be further seen as $\operatorname{SU}(2)$ non-abelian generalization of previous extended $\mathrm{U}(1)$ abelian Stueckelberg mechanism. Due to the passive roles of neutral Higgs and $Z^{\prime}$, in this work we focus our attentions mainly on $W^{\prime}$ and related physics. For physics related to $W^{\prime}$, the strongest low energy phenomenological constraints come from $W-W^{\prime}$ mixing, $K_{L}-K_{S}$ mass differences and related CP violation parameters. On the platform of our EWCL, we can explore these constraints in detail, transferring them to the constraints on parameters of our EWCL and CKM matrix elements for right hand fermions. We will find that some of these constraints such as mixings among different particles are universal, while others are model class dependent. It should be emphasized that our EWCL will only cover those underlying models which include massive $W^{\prime \pm}, Z^{\prime}$ and neutral Higgs as lowest new particles beyond those already discovered particles. For those models which include new particle with mass lighter than $W^{\prime}$ or new particle combining with discovered particle together forms an irreducible representation of $\mathrm{SU}(2)$ group [3], our EWCL do not cover the corresponding physics. We argue for this alternative situation, a separate EWCL can be built to describe it and this situation will be investigated elsewhere.

Within the range of our EWCL, a special type of models are left-right symmetric models []. 2] which explore the possibility of spontaneous parity violation. The EWCL for this kind models is built up by some of us in ref. [27] for the bosonic part and ref. [28] for the matter part. Since we are interested in the general description for $W^{\prime}$ and $Z^{\prime}$ physics, it is purpose of this paper to generalize the discussion in ref. [27, 28] to cover leftright non-symmetric models. For bosonic part of EWCL, no matter which kind of model involving $W^{\prime}$ and $Z^{\prime}$, since gauge bosons and corresponding Goldstone bosons are all in triplet of $\mathrm{SU}(2)_{2}$ group, their interactions then are fixed as those given in ref. [27]. While for matter part, our treatment can cover the following different arrangements for fermion representations [29]:

1. Left-right symmetric (LR) [1], 2]: Left hand fermions belong to doublet of $\operatorname{SU}(2)_{1}$ and singlet of $\mathrm{SU}(2)_{2}$; Right hand fermions belong to doublet of $\mathrm{SU}(2)_{2}$ and singlet of $\mathrm{SU}(2){ }_{1}$.
2. Leptophobic (LP): Left hand fermions belong to doublet of $\mathrm{SU}(2)_{1}$ and singlet of $\mathrm{SU}(2)_{2}$; Right hand quarks belong to doublet of $\mathrm{SU}(2)_{2}$ and singlet of $\mathrm{SU}(2)_{1}$; Right hand leptons belong to singlets of both $\mathrm{SU}(2)$ 's.
3. Hadrophobic (HP): Left hand fermions belong to doublet of $\operatorname{SU}(2)_{1}$ and singlet of $\mathrm{SU}(2)_{2}$; Right hand leptons belong to doublet of $\mathrm{SU}(2)_{2}$ and singlet of $\mathrm{SU}(2)_{1}$; Right hand quarks belong to singlets of both $\mathrm{SU}(2)$ 's.
4. Fermionphobic (FP) 29-31: Left hand fermions belong to doublet of $\operatorname{SU}(2)_{1}$ and singlet of $\mathrm{SU}(2)_{2}$; Right hand fermions belong to singlets of both $\mathrm{SU}(2)$ 's.
5. Ununified (UN) [4]: Left hand leptons belong to doublet of $\mathrm{SU}(2)_{1}$ and singlet of $\mathrm{SU}(2)_{2}$; Left hand quarks belong to doublet of $\mathrm{SU}(2)_{2}$ and singlet of $\mathrm{SU}(2)_{1}$; Right hand fermions belong to singlet of $\mathrm{SU}(2)_{1} \otimes \mathrm{SU}(2)_{2}$.
6. Non-universal (NU) 32): One or two special family left hand fermions (typical situation is the first two light families) belong to doublet of $\mathrm{SU}(2)_{1}$ and singlet of $\mathrm{SU}(2)_{2}$; Remaining left hand fermions belong to doublet of $\mathrm{SU}(2)_{2}$ and singlet of $\mathrm{SU}(2)_{1}$; Right hand fermions belong to singlet of $\mathrm{SU}(2)_{1} \otimes \mathrm{SU}(2)_{2}$.

In fact, the designation of group $\mathrm{SU}(2)_{1}$ and $\mathrm{SU}(2)_{2}$ may be arbitrary. If we identify the $\mathrm{SU}(2)_{1}$ with the $\mathrm{SU}(2)_{L}$ in the SM in the absence of mixing, it can be shown that the above 6 cases have covered all possible fermion $\mathrm{SU}(2)_{1} \otimes \mathrm{SU}(2)_{2}$ arrangements. The matter part EWCL given in ref. [28] only involves situation 1 in which although the arrangement of fermion representations is left-right symmetric, the couplings may or may not be left-right symmetric. Considering the fact that conventional EWCL formalism only deals with the system with particles fixed in some special group representations, the generalization of the expression to cover different fermion representation arrangements is not a trivial work.

This paper is organized as follows: section 2 is the introduction of a our EWCL which covers all above situations. For the bosonic part we accurate up to order of $p^{4}$. For matter part, we limit us in dimension three Yukawa type and dimension four gauge interaction terms. In section 3, we discuss mixings among $W-W^{\prime}$ and $A-Z-Z^{\prime}$ and introduce CKM matrix to diagonalize fermion mass matrix. Goldstone boson, Higgs boson and gauge boson couplings to quarks are given in section 4. We build up effective Hamiltonian for mixing of neutral $K$ and $B$ systems in section 5. In section 6, we discuss the constraints on our EWCL for LR and LP models from mass differences in $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}, B_{s}^{0}-\bar{B}_{s}^{0}$ systems and indirect CP violation parameter $\epsilon_{K}$. Section 7 is the summary.

## 2. EWCL in gauge eigenstates

We first introduce the bosonic part of EWCL which basically is the same as that for leftright symmetric models given in ref. [27. Let $B_{\mu}, W_{1, \mu}^{a}, W_{2, \mu}^{a}$ be electroweak gauge fields ( $a=1,2,3$ ) and two by two unitary unimodular matrices $U_{1}$ and $U_{2}$ be corresponding goldstone boson fields, $h$ be neutral Higgs field which is singlet of $\mathrm{SU}(2)_{1} \otimes \mathrm{SU}(2)_{2} \otimes \mathrm{U}(1)$ group. Consider covariant derivatives for goldstone fields $D_{\mu} U_{i}=\partial_{\mu} U_{i}+i g_{i} \frac{\tau^{a}}{2} W_{i, \mu}^{a} U_{i}-$ $i g U_{i} \frac{\tau_{3}}{2} B_{\mu}$ and building blocks $X_{i}^{\mu} \equiv U_{i}^{\dagger}\left(D^{\mu} U_{i}\right), \bar{W}_{i, \mu \nu} \equiv U_{i}^{\dagger} g_{i} W_{i, \mu \nu} U_{i}$ for $i=1,2$. The lowest order of chiral Lagrangian is the Higgs potential $\mathcal{L}_{0}=-V(h)$ and $p^{2}$ order of Lagrangian is

$$
\begin{align*}
\mathcal{L}_{2}= & \frac{1}{2}\left(\partial_{\mu} h\right)^{2}-\frac{1}{4} f_{1}^{2} \operatorname{tr}\left(X_{1, \mu} X_{1}^{\mu}\right)-\frac{1}{4} f_{2}^{2} \operatorname{tr}\left(X_{2, \mu} X_{2}^{\mu}\right)+\frac{1}{2} \kappa f_{1} f_{2} \operatorname{tr}\left(X_{1}^{\mu} X_{2}^{\mu}\right)  \tag{2.1}\\
& +\frac{1}{4} \beta_{1,1} f_{1}^{2}\left[\operatorname{tr}\left(\tau^{3} X_{1, \mu}\right)\right]^{2}+\frac{1}{4} \beta_{2,1} f_{2}^{2}\left[\operatorname{tr}\left(\tau^{3} X_{2, \mu}\right)\right]^{2}+\frac{1}{2} \tilde{\beta}_{1} f_{1} f_{2}\left[\operatorname{tr}\left(\tau^{3} X_{1, \mu}\right)\right]\left[\operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right)\right]
\end{align*}
$$

$p^{4}$ order Lagrangian can be divided into six parts,

$$
\begin{equation*}
\mathcal{L}_{4}=\mathcal{L}_{K}+\mathcal{L}_{1}+\mathcal{L}_{H 1}+\mathcal{L}_{2}+\mathcal{L}_{H 2}+\mathcal{L}_{C} \tag{2.2}
\end{equation*}
$$

with kinetic part of $p^{4}$ order Lagrangian $\mathcal{L}_{K}$

$$
\begin{equation*}
\mathcal{L}_{K}=-\frac{1}{4} W_{1, \mu \nu}^{a} W_{1}^{\mu \nu, a}-\frac{1}{4} W_{2, \mu \nu}^{a} W_{2}^{\mu \nu, a}-\frac{1}{4} B_{\mu \nu} B^{\mu \nu} \tag{2.3}
\end{equation*}
$$

$\mathcal{L}_{i}, i=1,2$ are terms of $p^{4}$ order Lagrangian which involve the gauge bosons of first(second) interaction group $\mathrm{SU}(2)_{1}\left(\mathrm{SU}(2)_{2}\right)$ without differential of higgs

$$
\begin{align*}
\mathcal{L}_{i}= & \frac{1}{2} \alpha_{i, 1} g B_{\mu \nu} \operatorname{tr}\left(\tau^{3} \bar{W}_{i}^{\mu \nu}\right)+i \alpha_{i, 2} g B_{\mu \nu} \operatorname{tr}\left(\tau^{3} X_{i}^{\mu} X_{i}^{\nu}\right)+2 i \alpha_{i, 3} \operatorname{tr}\left(\bar{W}_{i, \mu \nu} X_{i}^{\mu} X_{i}^{\nu}\right) \\
& +\alpha_{i, 4}\left[\operatorname{tr}\left(X_{i, \mu} X_{i, \nu}\right)\right]^{2}+\alpha_{i, 5}\left[\operatorname{tr}\left(X_{i, \mu}^{2}\right)\right]^{2}+\alpha_{i, 6} \operatorname{tr}\left(X_{i, \mu} X_{i, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{i}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{i}^{\nu}\right) \\
& +\alpha_{i, 7} \operatorname{tr}\left(X_{i, \mu}^{2}\right)\left[\operatorname{tr}\left(\tau^{3} X_{i, \nu}\right)\right]^{2}+\frac{1}{4} \alpha_{i, 8}\left[\operatorname{tr}\left(\tau^{3} \bar{W}_{i, \mu \nu}\right)\right]^{2}+i \alpha_{i, 9} \operatorname{tr}\left(\tau^{3} \bar{W}_{i, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} X_{i}^{\mu} X_{i}^{\nu}\right) \\
& +\frac{1}{2} \alpha_{i, 10}\left[\operatorname{tr}\left(\tau^{3} X_{i, \mu}\right) \operatorname{tr}\left(\tau^{3} X_{i, \nu}\right)\right]^{2}+\alpha_{i, 11} \epsilon^{\mu \nu \rho \lambda} \operatorname{tr}\left(\tau^{3} X_{i, \mu}\right) \operatorname{tr}\left(X_{i, \nu} \bar{W}_{i, \rho \lambda}\right) \\
& +2 \alpha_{i, 12} \operatorname{tr}\left(\tau^{3} X_{i, \mu}\right) \operatorname{tr}\left(X_{i, \nu} \bar{W}_{i}^{\mu \nu}\right)+\frac{1}{4} \alpha_{i, 13} g \epsilon^{\mu \nu \rho \sigma} B_{\mu \nu} \operatorname{tr}\left(\tau^{3} \bar{W}_{i, \rho \sigma}\right) \\
& +\frac{1}{8} \alpha_{i, 14} \epsilon^{\mu \nu \rho \sigma} \operatorname{tr}\left(\tau^{3} \bar{W}_{i, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} \bar{W}_{i, \rho \sigma}\right) . \tag{2.4}
\end{align*}
$$

$\mathcal{L}_{\mathrm{Hi}}, i=1,2$ are first(second) interaction group part of $p^{4}$ order Lagrangian with differential of Higgs

$$
\begin{align*}
\mathcal{L}_{\mathrm{Hi}}= & \left(\partial_{\mu} h\right)\left\{\bar{\alpha}_{H i, 1} \operatorname{tr}\left(\tau^{3} X_{i}^{\mu}\right) \operatorname{tr}\left(X_{i, \nu}^{2}\right)+\bar{\alpha}_{H i, 2} \operatorname{tr}\left(\tau^{3} X_{i}^{\nu}\right) \operatorname{tr}\left(X_{i}^{\mu} X_{i, \nu}\right)+\bar{\alpha}_{H i, 3} \operatorname{tr}\left(\tau^{3} X_{i}^{\nu}\right) \operatorname{tr}\left(\tau^{3} X_{i}^{\mu} X_{i, \nu}\right)\right. \\
& +\bar{\alpha}_{H i, 4} \operatorname{tr}\left(\tau^{3} X_{i}^{\mu}\right)\left[\operatorname{tr}\left(\tau^{3} X_{i, \nu}\right)\right]^{2}+i \bar{\alpha}_{H i, 5} \operatorname{tr}\left(\tau^{3} X_{i, \nu}\right) \operatorname{tr}\left(\tau^{3} \bar{W}_{i}^{\mu \nu}\right)+i g \bar{\alpha}_{H i, 6} B^{\mu \nu} \operatorname{tr}\left(\tau^{3} X_{i, \nu}\right) \\
& \left.+i \bar{\alpha}_{H i, 7} \operatorname{tr}\left(\tau^{3} \bar{W}_{i}^{\mu \nu} X_{i, \nu}\right)+i \bar{\alpha}_{H i, 8} \operatorname{tr}\left(\bar{W}_{i}^{\mu \nu} X_{i, \nu}\right)\right\}+\left(\partial_{\mu} h\right)\left(\partial_{\nu} h\right)\left[\bar{\alpha}_{H i, 9} \operatorname{tr}\left(\tau^{3} X_{i}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{i}^{\nu}\right)\right. \\
& \left.+\bar{\alpha}_{H i, 10} \operatorname{tr}\left(X_{i}^{\mu} X_{i}^{\nu}\right)\right]+\left(\partial_{\mu} h\right)^{2}\left\{\bar{\alpha}_{H i, 11}\left[\operatorname{tr}\left(\tau^{3} X_{i, \nu}\right)\right]^{2}+\bar{\alpha}_{H i, 12} \operatorname{tr}\left(X_{i, \nu}^{2}\right)\right\} \\
& +\bar{\alpha}_{H i, 13}\left(\partial_{\mu} h\right)^{2}\left(\partial_{\nu} h\right) \operatorname{tr}\left(\tau^{3} X_{i}^{\nu}\right)+\bar{\alpha}_{H i, 14}\left(\partial_{\mu} h\right)^{4} \tag{2.5}
\end{align*}
$$

The most complex interaction is the crossing part of $p^{4}$ order Lagrangian

$$
\begin{aligned}
& \mathcal{L}_{C}= i \tilde{\alpha}_{2} g B_{\mu \nu} \operatorname{tr}\left(\tau^{3} X_{1}^{\mu} X_{2}^{\nu}\right)+2 i \tilde{\alpha}_{3,1} \operatorname{tr}\left(\bar{W}_{1, \mu \nu} X_{2}^{\mu} X_{2}^{\nu}\right)+2 i \tilde{\alpha}_{3,2} \operatorname{tr}\left(\bar{W}_{2, \mu \nu} X_{1}^{\mu} X_{1}^{\nu}\right) \\
&+2 i \tilde{\alpha}_{3,3} \operatorname{tr}\left(\bar{W}_{1, \mu \nu} X_{1}^{\mu} X_{2}^{\nu}\right)+2 i \tilde{\alpha}_{3,4} \operatorname{tr}\left(\bar{W}_{2, \mu \nu}^{\mu} X_{2}^{\mu} X_{1}^{\nu}\right)+\tilde{\alpha}_{4,1} \operatorname{tr}\left(X_{1, \mu} X_{1, \nu}\right) \operatorname{tr}\left(X_{2}^{\mu} X_{2}^{\nu}\right) \\
&+\tilde{\alpha}_{4,2}\left[\operatorname{tr}\left(X_{1, \mu} X_{2, \nu}\right)\right]^{2}+\tilde{\alpha}_{4,3} \operatorname{tr}\left(X_{1, \mu} X_{2, \nu}\right) \operatorname{tr}\left(X_{2}^{\mu} X_{1}^{\nu}\right)+\tilde{\alpha}_{4,4} \operatorname{tr}\left(X_{1, \mu} X_{2, \nu}\right) \operatorname{tr}\left(X_{2}^{\mu} X_{2}^{\nu}\right) \\
&+\tilde{\alpha}_{4,5} \operatorname{tr}\left(X_{2, \mu} X_{1, \nu}\right) \operatorname{tr}\left(X_{1}^{\mu} X_{1}^{\nu}\right)+\tilde{\alpha}_{5,1} \operatorname{tr}\left(X_{1, \mu}^{2}\right) \operatorname{tr}\left(X_{2, \nu}^{2}\right)+\tilde{\alpha}_{5,2}\left[\operatorname{tr}\left(X_{1, \mu} X_{2}^{\mu}\right)\right]^{2} \\
&+\tilde{\alpha}_{5,3} \operatorname{tr}\left(X_{1, \mu} X_{2}^{\mu}\right) \operatorname{tr}\left(X_{2, \nu}^{2}\right)+\tilde{\alpha}_{5,4} \operatorname{tr}\left(X_{2, \mu} X_{1}^{\mu}\right) \operatorname{tr}\left(X_{1, \nu}^{2}\right) \\
&+\tilde{\alpha}_{6,1} \operatorname{tr}\left(X_{1, \mu} X_{1, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right)+\tilde{\alpha}_{6,2} \operatorname{tr}\left(X_{2, \mu} X_{2, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right) \\
&+\tilde{\alpha}_{6,3} \operatorname{tr}\left(X_{1, \mu} X_{2, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right)+\tilde{\alpha}_{6,4} \operatorname{tr}\left(X_{1, \mu} X_{2, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right) \\
&+\tilde{\alpha}_{6,5} \operatorname{tr}\left(X_{1, \mu} X_{2, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right)+\tilde{\alpha}_{6,6} \operatorname{tr}\left(X_{2, \mu} X_{1, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right) \\
&+\tilde{\alpha}_{6,7} \operatorname{tr}\left(X_{1, \mu} X_{1, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right)+\tilde{\alpha}_{6,8} \operatorname{tr}\left(X_{2, \mu} X_{2, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right) \\
&\left.+\tilde{\alpha}_{7,1} \operatorname{tr}\left(X_{1, \mu}^{2}\right)\left[\operatorname{tr}\left(\tau^{3} X_{2, \nu}\right)\right]^{2}+\tilde{\alpha}_{7,2} \operatorname{tr}\left(X_{2, \mu}^{2}\right) \operatorname{tr}\left(\tau^{3} X_{1, \nu}\right)\right]^{2} \\
&+\tilde{\alpha}_{7,3} \operatorname{tr}\left(X_{1, \mu} X_{2}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{1, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right)+\tilde{\alpha}_{7,4} \operatorname{tr}\left(X_{1, \mu} X_{2}^{\mu}\right)\left[\operatorname{tr}\left(\tau^{3} X_{2, \nu}\right)\right]^{2} \\
&+\tilde{\alpha}_{7,5} \operatorname{tr}\left(X_{2, \mu} X_{1}^{\mu}\right)\left[\operatorname{tr}\left(\tau^{3} X_{1, \nu}\right)\right]^{2}+\tilde{\alpha}_{7,6} \operatorname{tr}\left(X_{1, \mu}^{2}\right) \operatorname{tr}\left(\tau^{3} X_{1, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right) \\
&+\tilde{\alpha}_{7,7} \operatorname{tr}\left(X_{2, \mu}^{2} \operatorname{tr}\left(\tau^{3} X_{2, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right)+\frac{1}{4} \tilde{\alpha}_{8} \operatorname{tr}\left(\tau^{3} \bar{W}_{1, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} \bar{W}_{2}^{\mu \nu}\right)\right. \\
& \text { and }
\end{aligned}
$$

$$
\begin{align*}
& +i \tilde{\alpha}_{9,1} \operatorname{tr}\left(\tau^{3} \bar{W}_{1, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu} X_{2}^{\nu}\right)+i \tilde{\alpha}_{9,2} \operatorname{tr}\left(\tau^{3} \bar{W}_{2, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} X_{L}^{\mu} X_{L}^{\nu}\right) \\
& +i \tilde{\alpha}_{9,3} \operatorname{tr}\left(\tau^{3} \bar{W}_{1, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\mu} X_{2}^{\nu}\right)+i \tilde{\alpha}_{9,4} \operatorname{tr}\left(\tau^{3} \bar{W}_{2, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu} X_{1}^{\nu}\right) \\
& +\frac{1}{2} \tilde{\alpha}_{10,1}\left[\operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(\tau^{3} X_{2, \nu}\right)\right]^{2}+\frac{1}{2}\left[\tilde{\alpha}_{10,2}\left[\operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right)\right]^{2}\right. \\
& +\frac{1}{2} \tilde{\alpha}_{10,3} \operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right)\left[\operatorname{tr}\left(\tau^{3} X_{2, \nu}\right)\right]^{2}+\frac{1}{2} \tilde{\alpha}_{10,4} \operatorname{tr}\left(\tau^{3} X_{2, \mu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right)\left[\operatorname{tr}\left(\tau^{3} X_{1, \nu}\right)\right]^{2} \\
& +\tilde{\alpha}_{11,1} \epsilon^{\mu \nu \rho \lambda} \operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(X_{2, \nu} \bar{W}_{2, \rho \lambda}\right)+\tilde{\alpha}_{11,2} \epsilon^{\mu \nu \rho \lambda} \operatorname{tr}\left(\tau^{3} X_{2, \mu}\right) \operatorname{tr}\left(X_{1, \nu} \bar{W}_{1, \rho \lambda}\right) \\
& +\tilde{\alpha}_{11,3} \epsilon^{\mu \nu \rho \lambda} \operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(X_{1, \nu} \bar{W}_{2, \rho \lambda}\right)+\tilde{\alpha}_{11,4} \epsilon^{\mu \nu \rho \lambda} \operatorname{tr}\left(\tau^{3} X_{2, \mu}\right) \operatorname{tr}\left(X_{2, \nu} \bar{W}_{1, \rho \lambda}\right) \\
& +\tilde{\alpha}_{11,5} \epsilon^{\mu \nu \rho \lambda} \operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(X_{2, \nu} \bar{W}_{1, \rho \lambda}\right)+\tilde{\alpha}_{11,6} \epsilon^{\mu \nu \rho \lambda} \operatorname{tr}\left(\tau^{3} X_{2, \mu}\right) \operatorname{tr}\left(X_{1, \nu} \bar{W}_{2, \rho \lambda}\right) \\
& +2 \tilde{\alpha}_{12,1} \operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(X_{2, \nu} \bar{W}_{2}^{\mu \nu}\right)+2 \tilde{\alpha}_{12,2} \operatorname{tr}\left(\tau^{3} X_{2, \mu}\right) \operatorname{tr}\left(X_{1, \nu} \bar{W}_{1}^{\mu \nu}\right) \\
& +2 \tilde{\alpha}_{12,3} \operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(X_{1, \nu} \bar{W}_{2}^{\mu \nu}\right)+2 \tilde{\alpha}_{12,4} \operatorname{tr}\left(\tau^{3} X_{2, \mu}\right) \operatorname{tr}\left(X_{2, \nu} \bar{W}_{1}^{\mu \nu}\right) \\
& +2 \tilde{\alpha}_{12,5} \operatorname{tr}\left(\tau^{3} X_{1, \mu}\right) \operatorname{tr}\left(X_{2, \nu} \bar{W}_{1}^{\mu \nu}\right)+2 \tilde{\alpha}_{12,6} \operatorname{tr}\left(\tau^{3} X_{2, \mu}\right) \operatorname{tr}\left(X_{1, \nu} \bar{W}_{2}^{\mu \nu}\right) \\
& +\frac{1}{8} \tilde{\alpha}_{14} \epsilon^{\mu \nu \rho \sigma} \operatorname{tr}\left(\tau^{3} \bar{W}_{1, \mu \nu}\right) \operatorname{tr}\left(\tau^{3} \bar{W}_{2, \rho \sigma}\right)+\tilde{\alpha}_{15} \epsilon^{\mu \nu \rho \sigma} \operatorname{tr}\left(\bar{W}_{1, \mu \nu} \bar{W}_{2, \rho \sigma}\right) \\
& +\left(\partial_{\mu} h\right)\left\{\tilde{\alpha}_{H, 1,1} \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right) \operatorname{tr}\left(X_{1, \nu}^{2}\right)+\tilde{\alpha}_{H, 1,2} \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right) \operatorname{tr}\left(X_{2, \nu}^{2}\right)\right. \\
& +\tilde{\alpha}_{H, 2,1} \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right) \operatorname{tr}\left(X_{1}^{\mu} X_{L, \nu}\right)+\tilde{\alpha}_{H, 2,2} \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right) \operatorname{tr}\left(X_{2}^{\mu} X_{2, \nu}\right) \\
& +\tilde{\alpha}_{H, 3,1} \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\mu} X_{1, \nu}\right)+\tilde{\alpha}_{H, 3,2} \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\mu} X_{2, \nu}\right) \\
& +\tilde{\alpha}_{H, 4,1} \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right)\left[\operatorname{tr}\left(\tau^{3} X_{1, \nu}\right)\right]_{2}^{2}+\tilde{\alpha}_{H, 4,2} \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right) \operatorname{tr}\left(\tau^{3} X_{1, \nu}\right) \\
& +\tilde{\alpha}_{H, 4,3} \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right) \operatorname{tr}\left(\tau^{3} X_{2, \nu}\right)+\tilde{\alpha}_{H, 4,4} \operatorname{tr}\left(\tau^{3} X_{1}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right) \operatorname{tr}\left(\tau^{3} X_{2, \nu}\right) \\
& \left.+i \tilde{\alpha}_{H, 5,1} \operatorname{tr}\left(\tau^{3} X_{2, \nu}\right) \operatorname{tr}\left(\tau^{3} \bar{W}_{1}^{\mu \nu}\right)+i \tilde{\alpha}_{H, 5,2} \operatorname{tr}\left(\tau^{3} X_{1, \nu}\right) \operatorname{tr}\left(\tau^{3} \bar{W}_{2}^{\mu \nu}\right)\right\} \\
& \left.+\left(\partial_{\mu} h\right)\left(\partial_{\nu} h\right) \tilde{\alpha}_{H, 9} \operatorname{tr}\left(\tau^{3} X_{2}^{\mu}\right) \operatorname{tr}\left(\tau^{3} X_{1}^{\nu}\right)+\left(\partial_{\mu} h\right)^{2} \tilde{\alpha}_{H, 11} \operatorname{tr}\left(\tau^{3} X_{1, \nu}\right) \operatorname{tr}\left(\tau^{3} X_{2}^{\nu}\right)\right] \tag{2.6}
\end{align*}
$$

Above interaction terms already include all possible $p^{4}$ order CP-conserving and CPviolating operators and all $\alpha$ coefficients are functions of higgs field $h$.

Now we come to matter part of EWCL. Except gauge and goldstone fields introduced in bosonic part EWCL, matter part EWCL further involves fermions which include SM quarks and leptons (three generation right hand neutrinos are introduced in our theory, no other sterile neutrinos are included in). We denote them by left and right hand quark and lepton doublets $q_{\alpha L, R}$ and $l_{\alpha L, R}$ with generation index $\alpha$ being summed over the quark and lepton flavors. The various models defined by the transformation properties of their fermion contents with respect to the gauge group are summarized in table I.

Since above fermions can belong to different representations for different underlying models, an universal expression to cover all these possible arrangements is needed. To reach this aim, we introduce two goldstone operators $\hat{U}_{L}$ and $\hat{U}_{R}$ by defining their arbitrary

| Fields/Models | LR | LP | HP | FP | UN | NU |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $q_{\alpha L}=\binom{u_{\alpha L}}{d_{\alpha L}}$ | $\left(2,1, \frac{1}{6}\right)$ | $\left(2,1, \frac{1}{6}\right)$ | $\left(2,1, \frac{1}{6}\right)$ | $\left(2,1, \frac{1}{6}\right)$ | $\left(1,2, \frac{1}{6}\right)$ | $\left(2,1, \frac{1}{6}\right) \delta_{\alpha \alpha_{1}}+\left(1,2, \frac{1}{6}\right) \delta_{\alpha \alpha_{2}}$ |
| $q_{\alpha R}=\binom{u_{\alpha R}}{d_{\alpha R}}$ | $\left(1,2, \frac{1}{6}\right)$ | $\left(1,2, \frac{1}{6}\right)$ | $\left(1,1, \frac{2}{3}\right)$ | $\left(1,1, \frac{2}{3}\right)$ | $\left(1,1, \frac{2}{3}\right)$ | $\left(1,1, \frac{2}{3}\right)$ |
| $l_{\alpha L}=\binom{\nu_{\alpha L}}{e_{\alpha L}^{-}}$ | $\left(2,1,-\frac{1}{2}\right)$ | $\left(2,1,-\frac{1}{2}\right)$ | $\left(2,1,-\frac{1}{2}\right)$ | $\left(2,1,-\frac{1}{2}\right)$ | $\left(1,1,-\frac{1}{3}\right)$ | $\left(1,1,-\frac{1}{2}\right)$ |
| $l_{\alpha R}=\binom{\nu_{\alpha R}}{e_{\alpha R}^{-}}$ | $\left(1,2,-\frac{1}{3}\right)$ |  |  |  |  |  |
| $\left(\begin{array}{ll}\left(1,-\frac{1}{2}\right) & \delta_{\alpha \alpha_{1}}+\left(1,2,-\frac{1}{2}\right) \delta_{\alpha \alpha_{2}} \\ \hline\end{array}\right.$ | $(1,-1)$ | $\left(1,2,-\frac{1}{2}\right)$ | - | - | - |  |

Table 1: Fermion transformation properties for different models considered in the text. The numbers in brackets refer to $\mathrm{SU}(2)_{1}, \mathrm{SU}(2)_{2}$ and $\mathrm{U}(1)$, respectively. Color indices are implicit. The right hand neutrinos are not present in some of original models LP, FP, UN and NU labeled by -. Including them in this work is harmless to these models and their representation is $(1,1,0)$.
function $f\left(\hat{U}_{R}, \hat{U}_{L}, D_{\mu} \hat{U}_{R}, D_{\mu} \hat{U}_{L}\right)$ action on fermion field as
$f\left(\hat{U}_{R}, \hat{U}_{L}, D_{\mu} \hat{U}_{R}, D_{\mu} \hat{U}_{L}\right) q_{\alpha}= \begin{cases}f\left(U_{2}, U_{1}, D_{\mu} U_{2}, D_{\mu} U_{1}\right) q_{\alpha} & \text { LR } \\ f\left(U_{2}, U_{1}, D_{\mu} U_{2}, D_{\mu} U_{1}\right) q_{\alpha} & \text { LP } \\ f\left(1, U_{1}, 0, D_{\mu} U_{1}\right) q_{\alpha} & \text { HP } \\ f\left(1, U_{1}, 0, D_{\mu} U_{1}\right) q_{\alpha} & \text { FP } \\ f\left(1, U_{2}, 0, D_{\mu} U_{2}\right) q_{\alpha} & \text { UN } \\ f\left(1, U_{1}, 0, D_{\mu} U_{1}\right) q_{\alpha} \delta_{\alpha \alpha_{1}}+f\left(1, U_{2}, 0, D_{\mu} U_{2}\right) q_{\alpha} \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases}$
$f\left(\hat{U}_{R}, \hat{U}_{L}, D_{\mu} \hat{U}_{R}, D_{\mu} \hat{U}_{L}\right) l_{\alpha}= \begin{cases}f\left(U_{2}, U_{1}, D_{\mu} U_{2}, D_{\mu} U_{1}\right) l_{\alpha} & \text { LR } \\ f\left(1, U_{1}, 0, D_{\mu} U_{1}\right) l_{\alpha} & \text { LP } \\ f\left(U_{2}, U_{1}, D_{\mu} U_{2}, D_{\mu} U_{1}\right) l_{\alpha} & \mathrm{HP} \\ f\left(1, U_{1}, 0, D_{\mu} U_{1}\right) l_{\alpha} & \mathrm{FP} \\ f\left(1, U_{1}, 0, D_{\mu} U_{1}\right) l_{\alpha} & \mathrm{UN} \\ f\left(1, U_{1}, 0, D_{\mu} U_{1}\right) l_{\alpha} \delta_{\alpha \alpha_{1}}+f\left(1, U_{2}, 0, D_{\mu} U_{2}\right) l_{\alpha} \delta_{\alpha \alpha_{2}} & \mathrm{NU}\end{cases}$
where in the case of "Non-universality generation", $\alpha_{1}$ denote the specified generation (typically first two generations) which acts as doublet of $\mathrm{SU}(2)_{1}$ and singlet of $\mathrm{SU}(2)_{2} ; \alpha_{2}$ denote the remaining generation which acts as doublet of $\mathrm{SU}(2)_{2}$ and singlet of $\mathrm{SU}(2)_{1}$.

With help of above representations, we now can write down the universal dimension three Yukawa type interactions. For lepton part,

$$
\begin{equation*}
\mathcal{L}_{\mathrm{Y}, \text { lepton }}=\bar{l}_{\alpha L}^{I}\left[\hat{U}_{L}\left(y^{\alpha \beta}+y_{3}^{\alpha \beta} \tau^{3}\right) \hat{U}_{R}^{\dagger}\right] l_{\beta R}^{I}+\frac{1}{2}\left[h_{L}^{\alpha \beta} \overline{l_{\alpha L}^{\mathrm{Ic}}} \hat{U}_{L}^{*}\left(1+\tau^{3}\right) \hat{U}_{L}^{\dagger} l_{\beta L}^{I}+(L \rightarrow R)\right]+\text { h.c. } \tag{2.8}
\end{equation*}
$$

where $h_{L, R}^{\alpha \beta}$ are hermitian functions of Higgs field $h . l^{c}=C \bar{l}^{T}$ is charge conjugate field of $l$ with $C$ being the charge conjugation matrix. Symbol "I" indicates that they are gauge eigenstates. For quark part,

$$
\begin{equation*}
\mathcal{L}_{\mathrm{Y}, \text { quark }}=\bar{q}_{\alpha L}^{I}\left[\hat{U}_{L}\left(\tau^{u} y_{u}^{\alpha \beta}+\tau^{d} y_{d}^{\alpha \beta}\right) \hat{U}_{R}^{\dagger}\right] q_{\beta R}^{I}+\text { h.c. } \tag{2.9}
\end{equation*}
$$

where $\tau^{u}=\frac{1+\tau^{3}}{2}$ and $\tau^{d}=\frac{1-\tau^{3}}{2}$. Coefficients $y_{u}^{\alpha \beta}, y_{d}^{\alpha \beta}$ are functions of Higgs field.

The next is dimension four gauge interaction part Lagrangian

$$
\begin{align*}
& \mathcal{L}_{f-4}=i \sum_{\alpha}\left\{\bar{q}_{\alpha L}^{I} \not D q_{\alpha L}^{I}+\delta_{L, 1, \alpha} \bar{q}_{\alpha L}^{I} \hat{U}_{L}\left(\not D \hat{U}_{L}\right)^{\dagger} q_{\alpha L}^{I}+\delta_{L, 2, \alpha} \bar{q}_{\alpha R}^{I} \hat{U}_{R} \hat{U}_{L}^{\dagger}\left(\not D \hat{U}_{L}\right) \hat{U}_{R}^{\dagger} q_{\alpha R}^{I}\right. \\
& +\delta_{L, 3, \alpha} \bar{q}_{\alpha L}^{I}\left[\left(\not D \hat{U}_{L}\right) \tau^{3} \hat{U}_{L}^{\dagger}-\hat{U}_{L} \tau^{3}\left(\not D \hat{U}_{L}\right)^{\dagger}\right] q_{\alpha L}^{I}+\delta_{L, 4, \alpha} \bar{q}_{\alpha L}^{I} \hat{U}_{L} \tau^{3} \hat{U}_{L}^{\dagger}\left(\not D \hat{U}_{L}\right) \tau^{3} \hat{U}_{L}^{\dagger} q_{\alpha L}^{I} \\
& +\delta_{L, 5, \alpha} \bar{q}_{\alpha R}^{I} \hat{U}_{R}\left[\tau^{3} \hat{U}_{L}^{\dagger}\left(\not D \hat{U}_{L}\right)-\left(\not D \hat{U}_{L}\right)^{\dagger} \hat{U}_{L} \tau^{3}\right] \hat{U}_{R}^{\dagger} q_{\alpha R}^{I}+\delta_{L, 6, \alpha} \bar{q}_{\alpha R}^{I} \hat{U}_{R} \tau^{3} \hat{U}_{L}^{\dagger}\left(\not D \hat{U}_{L}\right) \tau^{3} \hat{U}_{R}^{\dagger} q_{\alpha R}^{I} \\
& \left.+\delta_{L, 7, \alpha}\left[\bar{q}_{\alpha L}^{I} \hat{U}_{L} \tau^{3} \hat{U}_{L}^{\dagger} \not D q_{\alpha L}^{I}-\left(\bar{q}_{\alpha L}^{I} \not D^{\dagger}\right) \hat{U}_{L} \tau^{3} \hat{U}_{L}^{\dagger} q_{\alpha L}^{I}\right]\right\}+q^{I} \rightarrow l^{I}, \delta \rightarrow \delta^{l}+L \leftrightarrow R, \tag{2.10}
\end{align*}
$$

in which

$$
D_{\mu} q_{\alpha}=\left\{\begin{array}{lc}
\left(\partial_{\mu}+i g_{1} \frac{\tau^{a}}{2} W_{1, \mu}^{a} P_{L}+i g_{2} \frac{\tau^{a}}{2} W_{2, \mu}^{a} P_{R}+\frac{i}{6} g B_{\mu}\right) q_{\alpha} & \text { LR, LP }  \tag{2.11}\\
\left(\partial_{\mu}+i g_{1} \frac{\tau^{a}}{2} W_{1, \mu}^{a} P_{L}+i g \frac{\tau^{\tau^{2}}}{2} B_{\mu} P_{R}+\frac{i}{6} g B_{\mu}\right) q_{\alpha} & \text { HP, FP } \\
\left(\partial_{\mu}+i g_{2} \frac{\tau^{a}}{2} W_{2, \mu}^{a} P_{L}+i g \frac{\tau^{3}}{2} B_{\mu} P_{R}+\frac{i}{6} g B_{\mu}\right) q_{\alpha} & \text { UN } \\
\left(\partial_{\mu}+i \delta_{\alpha \alpha_{1}} g_{1} \frac{\tau^{a}}{2} W_{1, \mu}^{a} P_{L}+i \delta_{\alpha \alpha_{2}} g_{2} \frac{\tau^{a}}{2} W_{2, \mu}^{a} P_{L}+i g \frac{\tau^{3}}{2} B_{\mu} P_{R}+\frac{i}{6} g B_{\mu}\right) q_{\alpha} & \text { NU }
\end{array}\right.
$$

$D_{\mu} l_{\alpha}= \begin{cases}\left(\partial_{\mu}+i g_{1} \frac{\tau^{a}}{2} W_{1, \mu}^{a} P_{L}+i g_{2} \frac{\tau^{a}}{2} W_{2, \mu}^{a} P_{R}-\frac{i}{2} g B_{\mu}\right) l_{\alpha} & \text { LR, HP } \\ \left(\partial_{\mu}+i g_{1} \frac{\tau^{a}}{2} W_{1, \mu}^{a} P_{L}+i g \frac{\tau^{3}}{2} B_{\mu} P_{R}-\frac{i}{2} g B_{\mu}\right) l_{\alpha} & \text { LP, FP, UN } \\ \left(\partial_{\mu}+i \delta_{\alpha \alpha_{1}} g_{1} \frac{\tau^{a}}{2} W_{1, \mu}^{a} P_{L}+i \delta_{\alpha \alpha_{2}} g_{2} \frac{\tau^{a}}{2} W_{2, \mu}^{a} P_{L}+i g \frac{\tau^{3}}{2} B_{\mu} P_{R}-\frac{i}{2} g B_{\mu}\right) l_{\alpha} & \text { NU }\end{cases}$
and $P_{R}=\left(1 \pm \gamma_{5}\right) / 2 .\left(\not D \hat{U}_{i}\right)^{\dagger} \equiv \gamma^{\mu}\left(D_{\mu} \hat{U}_{i}\right)^{\dagger}$ for $i=L, R$. In (2.10), coefficients $\delta$ and $\delta^{l}$ in general depend on generation indices which was not considered in original LR case in ref. 28.

## 3. EWCL in mass eigenstates

EWCL presented in last section is on the basis of gauge eigenstates. In this section, we diagonalize them to the basis of mass eigenstates. We will find that this diagonalization is universal for either boson sector or fermion sectors.

We first discuss boson sector. This part is the same as that in LR case [27], so we just list down the result. With convention $W_{i, \mu}^{ \pm}=\frac{1}{\sqrt{2}}\left(W_{i, \mu}^{1} \mp i W_{i, \mu}^{2}\right), i=1,2$, the mass terms in our bosonic part EWCL is

$$
\begin{align*}
\mathcal{L}_{M}= & \frac{1}{4} f_{1}^{2} g_{1}^{2} W_{1, \mu}^{+} W_{1}^{-, \mu}+\frac{1}{4} f_{2}^{2} g_{2}^{2} W_{2, \mu}^{+} W_{2}^{-, \mu}-\frac{1}{2} \kappa f_{1} f_{2} g_{1} g_{2}\left(W_{1, \mu}^{+} W_{2}^{-, \mu}+W_{2, \mu}^{+} W_{1}^{-, \mu}\right) \\
& +\frac{1}{8}\left(1-2 \beta_{1,1}\right) f_{1}^{2}\left(g_{1} W_{1, \mu}^{3}-g B_{\mu}\right)^{2}+\frac{1}{8}\left(1-2 \beta_{2,1}\right) f_{2}^{2}\left(g_{2} W_{2, \mu}^{3}-g B_{\mu}\right)^{2} \\
& -\frac{1}{4}\left(\kappa+2 \tilde{\beta}_{1}\right) f_{1} f_{2}\left(g_{1} W_{1, \mu}^{3}-g B_{\mu}\right)\left(g_{2} W_{2}^{3, \mu}-g B^{\mu}\right) \tag{3.1}
\end{align*}
$$

The charged and neutral gauge bosons are diagonalized through rotations

$$
\binom{W_{1}^{ \pm}}{W_{2}^{ \pm}}=\left(\begin{array}{cc}
\cos \zeta & -\sin \zeta  \tag{3.2}\\
\sin \zeta & \cos \zeta
\end{array}\right)\binom{W^{ \pm}}{W^{\prime \pm}} \quad\left(\begin{array}{c}
W_{1, \mu}^{3} \\
W_{2, \mu}^{3} \\
B_{\mu}
\end{array}\right)=\left(\begin{array}{lll}
x_{1} & x_{2} & x_{3} \\
y_{1} & y_{2} & y_{3} \\
v_{1} & v_{2} & v_{3}
\end{array}\right)\left(\begin{array}{c}
Z_{\mu} \\
Z_{\mu}^{\prime} \\
A_{\mu}
\end{array}\right)
$$

with mixing parameters given by

$$
\tan 2 \zeta=\frac{2 \kappa f_{1} f_{2} g_{1} g_{2}}{f_{2}^{2} g_{2}^{2}-f_{1}^{2} g_{1}^{2}} \quad\left(\begin{array}{lll}
x_{1} & x_{2} & x_{3}  \tag{3.3}\\
y_{1} & y_{2} & y_{3} \\
v_{1} & v_{2} & v_{3}
\end{array}\right)=V \Lambda \tilde{V}
$$

and

$$
\begin{aligned}
& \left(\begin{array}{ccc}
1-\alpha_{1,8} g_{1}^{2} & -\frac{1}{2} \tilde{\alpha}_{8} g_{1} g_{2}-\alpha_{1,1} g_{1} g \\
-\frac{1}{2} \tilde{\alpha}_{8} g_{1} g_{2} & 1-\alpha_{1,8} g_{2}^{2} & -\alpha_{2,1} g_{2} g \\
-\alpha_{1,1} g_{1} g & -\alpha_{2,1} g_{2} g & 1
\end{array}\right)=V\left(\begin{array}{ccc}
\lambda_{+} & 0 & 0 \\
0 & \lambda_{-} & 0 \\
0 & 0 & 1
\end{array}\right) V^{T}, \\
& \lambda_{ \pm}=1-\frac{1}{2} \alpha_{1,8} g_{1}^{2}-\frac{1}{2} \alpha_{2,8} g_{2}^{2} \pm\left[\alpha_{1,1}^{2} g_{1}^{2} g^{2}+\alpha_{2,1}^{2} g_{2}^{2} g^{2}+\frac{1}{4} \tilde{\alpha}_{8}^{2} g_{1}^{2} g_{2}^{2}+\frac{1}{4}\left(\alpha_{1,8} g_{1}^{2}-\alpha_{2,8} g_{2}^{2}\right)^{2}\right]^{1 / 2} . \\
& \Lambda V^{T} \tilde{M}_{0}^{2} V \Lambda=\tilde{V}\left(\begin{array}{ccc}
M_{Z}^{2} & 0 & 0 \\
0 & M_{Z^{\prime}}^{2} & 0 \\
0 & 0 & 0
\end{array}\right) \tilde{V}^{T}, \quad \Lambda \equiv\left(\begin{array}{ccc}
\frac{1}{\sqrt{\lambda+}} & 0 & 0 \\
0 & \frac{1}{\sqrt{\lambda-}} 0 \\
0 & 0 & 1
\end{array}\right), \\
& \tilde{M}_{0}^{2}=\left(\begin{array}{ccc}
\frac{1}{4}\left(1-2 \beta_{L, 1}\right) f_{1}^{2} g_{1}^{2} & -\frac{1}{4}\left(\kappa+2 \tilde{\beta}_{1}\right) f_{1} f_{2} g_{1} g_{2} & {\left[\left(2 \beta_{1,1}-1\right) f_{1}+\left(\kappa+2 \tilde{\beta}_{1}\right) f_{2}\right] \frac{f_{1} g_{1} g}{4}} \\
-\frac{1}{4}\left(\kappa+2 \tilde{\beta}_{1}\right) f_{1} f_{2} g_{1} g_{2} & \frac{1}{4}\left(1-2 \beta_{2,1}\right) f_{2}^{2} g_{2}^{2} & {\left[\left(2 \beta_{2,1}-1\right) f_{2}+\left(\kappa+2 \tilde{\beta}_{1}\right) f_{1}\right] \frac{f_{2} g_{2 g} g}{4}} \\
{\left[\left(2 \beta_{1,1}-1\right) f_{1}+\left(\kappa+2 \tilde{\beta}_{1}\right) f_{2}\right] \frac{f_{1} g_{11} g}{4}\left[\left(2 \beta_{2,1}-1\right) f_{2}+\left(\kappa+2 \tilde{\beta}_{1}\right) f_{1}\right] \frac{f_{2} g_{2 g} g}{4}} & {\left[\frac{1-2 \beta_{1,1}}{2} f_{1}^{2}+\frac{1-2 \beta_{2,1}}{2} f_{2}^{2}-\left(\kappa+2 \tilde{\beta}_{1}\right) f_{1} f_{2}\right] \frac{g_{2}^{2}}{2}}
\end{array}\right) .
\end{aligned}
$$

The results of gauge boson masses become

$$
\begin{align*}
M_{W}^{2}= & \frac{1}{4}\left[f_{1}^{2} g_{1}^{2}+f_{2}^{2} g_{2}^{2}-\sqrt{\left(f_{1}^{2} g_{1}^{2}-f_{2}^{2} g_{2}^{2}\right)^{2}+4 \kappa^{2} f_{1}^{2} f_{2}^{2} g_{1}^{2} g_{2}^{2}}\right], \\
M_{W^{\prime}}^{2}= & \frac{1}{4}\left[f_{1}^{2} g_{1}^{2}+f_{2}^{2} g_{2}^{2}+\sqrt{\left(f_{1}^{2} g_{1}^{2}-f_{2}^{2} g_{2}^{2}\right)^{2}+4 \kappa^{2} f_{1}^{2} f_{2}^{2} g_{1}^{2} g_{2}^{2}}\right]  \tag{3.4}\\
M_{Z}^{2}= & \frac{f_{1}^{2}\left[\left(1-\beta_{1,1}\right)\left(1-\beta_{2,1}\right)-\left(\kappa+\tilde{\beta}_{1}\right)^{2}\right]}{2\left(1-\beta_{2,1}\right)\left(g_{2}^{2}+g^{2}\right)} \times\left[g_{2}^{2} g^{2}+g_{1}^{2} g_{2}^{2}+g_{1}^{2} g^{2}\right. \\
& \left.-2 \alpha_{1,1}\left(g_{2}^{2}+g^{2}\right) g_{1}^{2} g_{2}^{2} g^{2}+2 g_{2}^{4} g^{4} \alpha_{2,1}+\alpha_{1,8} g_{1}^{4}\left(g_{2}^{2}+g^{2}\right)^{2}+\alpha_{2,8} g_{2}^{4} g^{4}-\alpha_{8}\left(g_{2}^{2}+g^{2}\right) g_{1}^{2} g_{2}^{2} g^{2}\right] \\
M_{Z^{\prime}}^{2}= & \frac{1}{2}\left[\left(1-\beta_{2,1}\right) f_{2}^{2}\left(g_{2}^{2}+g^{2}\right)-2 f_{1} f_{2} g^{2}\left(\kappa+\beta_{1}\right)+\left(1-\beta_{1,1}\right) f_{1}^{2}\left(g_{1}^{2}+g^{2}\right)\right] \\
& -\alpha_{1,1}^{2} g^{2} g_{1}^{2}\left[f_{1}^{2}\left(1-\beta_{1,1}\right)-f_{1} f_{2}\left(\kappa+\beta_{1}\right)\right]-\alpha_{2,1} g^{2} g_{2}^{2}\left[f_{2}^{2}\left(1-\beta_{2,1}\right)-f_{1} f_{2}\left(\kappa+\beta_{1}\right)\right] \\
& +\frac{1}{2} \alpha_{1,8} g_{1}^{4} f_{1}^{2}\left(1-\beta_{1,1}\right)+\frac{1}{2} \alpha_{2,8} g_{2}^{4} f_{2}^{2}\left(1-\beta_{2,1}\right)-\frac{1}{2} \alpha_{8} g_{1}^{2} g_{2}^{2} f_{1} f_{2}\left(\kappa+\beta_{1}\right)-M_{Z}^{2} .
\end{align*}
$$

For the gauge boson part, the most stringent constraint comes from $W-W^{\prime}$ mixing which is characterized by the mixing angle $\zeta$. Fortunately (3.3) tells us that this angle depends on two independent parameters $x \equiv \frac{f_{1} g_{1}}{f_{2} g_{2}}$ and $\kappa$. While (3.4) indicates that the ratio of $W$ and $W^{\prime}$ mass depends also on these two parameters, we just have two parameters $x$ and $\kappa$ to describe two physical quantities $\zeta$ and $M_{W} / M_{W^{\prime}}$ at this stage of effective Lagrangian. We can tune this two parameters making the mixing angle $\zeta$ be small enough to match experiment data and at the same time keeping the $W^{\prime}$ mass be in arbitrary values.

In any of candidate models, only those with very small $\kappa$ or small $x$ are phenomenologically allowed.

Next, we discuss fermion sector which includes lepton and quark parts. For lepton part, in unitary gauge, (2.8) become

$$
\begin{align*}
\left.\mathcal{L}_{\mathrm{Y}, \text { lepton }}\right|_{\text {Unitary gauge }} & =\mathcal{L}_{\mathrm{Me}}+\mathcal{L}_{M \nu}  \tag{3.5}\\
\mathcal{L}_{\mathrm{Me}} & =\overline{e^{-I}}{ }_{\alpha L}\left(y^{\alpha \beta}-y_{3}^{\alpha \beta}\right) e_{\beta R}^{-I}+\overline{e^{-I}}{ }_{\alpha R}\left(y^{\dagger \alpha \beta}-y_{3}^{\dagger \alpha \beta}\right) e_{\beta L}^{-I},  \tag{3.6}\\
\mathcal{L}_{M \nu} & =\bar{\nu}^{\bar{I}}{ }_{\alpha L}\left(y^{\alpha \beta}+y_{3}^{\alpha \beta}\right) \nu_{\beta R}^{I}+h_{L}^{\alpha \beta} \overline{\nu_{\alpha L}^{I \mathrm{c}}} \nu_{\beta L}^{I}+h_{R}^{\alpha \beta} \bar{\nu}_{\alpha R}^{\mathrm{Ic}} \nu_{\beta R}^{I}+\text { h.c. }, \tag{3.7}
\end{align*}
$$

For electron part, rotating the gauge eigenstates into the mass eigenstates with unitary matrices $\tilde{V}^{e}$ by $e_{L, R}^{-}=\tilde{V}_{L, R}^{e} e_{L, R}^{-I}$, we can reduce (3.6) to

$$
\begin{equation*}
\mathcal{L}_{\mathrm{Me}}={\overline{e^{-}}}_{L} \mathbf{M}^{e} e_{R}^{-}+{\overline{e^{-}}}_{R} \mathbf{M}^{e \dagger} e_{L}^{-} \tag{3.8}
\end{equation*}
$$

with diagonal mass matrix $\mathbf{M}^{e}=\tilde{V}_{L}^{e}\left(y-y_{3}\right) \tilde{V}_{R}^{e \dagger}$. For neutrino part, with help of relation $\overline{\nu_{1}^{c}} \nu_{2}^{c}=\overline{\nu_{2}} \nu_{1}$, we find coefficient matrices $h_{L}^{\alpha \beta}$ and $h_{R}^{\alpha \beta}$ can be chosen to be symmetric, then neutrino part of Lagrangian can be written as

$$
\mathcal{L}_{M \nu}=\frac{1}{2}\left(\overline{\nu_{L}^{I}} \overline{\nu_{R}^{\text {Ic }}}\right)\left(\begin{array}{cc}
2 h_{L} & y+y_{3}  \tag{3.9}\\
\left(y+y_{3}\right)^{T} & 2 h_{R}
\end{array}\right)\binom{\nu_{L}^{\text {Ic }}}{\nu_{R}^{I}}+\text { h.c. } .
$$

where $\nu_{L}^{\mathrm{Ic}}=\left(\begin{array}{c}\nu_{e}^{\mathrm{Ic}} \\ \nu_{\nu}^{\mathrm{Ic}} \\ \nu_{\tau}^{\mathrm{Ic}}\end{array}\right)_{L}$ is left-handed neutrino gauge eigenstates and $\nu_{R}^{I}=\left(\begin{array}{c}\nu_{e}^{I} \\ \nu_{\nu}^{I} \\ \nu_{\tau}^{I}\end{array}\right)_{R}$ is right-handed neutrino gauge eigenstates. The overall $6 \times 6$ neutrino mass matrix $\left(\begin{array}{cc}2 h_{L} & y+y_{3} \\ \left(y+y_{3}\right)^{T} & 2 h_{R}\end{array}\right)$ is symmetric and can be diagonalized by a unitary transformation,

$$
\left(\begin{array}{cc}
V & R  \tag{3.10}\\
S & U
\end{array}\right)^{\dagger}\left(\begin{array}{cc}
2 h_{L} & y+y_{3} \\
\left(y+y_{3}\right)^{T} & 2 h_{R}
\end{array}\right)\left(\begin{array}{cc}
V & R \\
S & U
\end{array}\right)^{*}=\left(\begin{array}{cc}
\hat{M}_{\nu} & 0 \\
0 & \hat{M}_{N}
\end{array}\right)
$$

where $\hat{M}_{\nu}=\operatorname{diag}\left\{m_{1}, m_{2}, m_{3}\right\}$ and $\hat{M}_{N}=\operatorname{diag}\left\{M_{1}, M_{2}, M_{3}\right\}$ with $m_{i}$ and $M_{i}$ (for $i=1,2,3)$ the light and heavy neutrino masses, respectively. $V$ is the $3 \times 3$ Maki-NakagawaSakata (MNS) neutrino mixing matrix [33] responsible for neutrino oscillations, and $R, S, U$ are all $3 \times 3$ matrices. After this diagonalization, one may express the neutrino gauge eigenstates $\nu_{\alpha}^{I}$ (for $\alpha=e, \mu, \tau$ ) in terms of the light and heavy neutrino mass states $\nu_{\alpha}$ and $N_{\alpha}$ :

$$
\left(\begin{array}{c}
\nu_{e}^{I}  \tag{3.11}\\
\nu_{\nu}^{I} \\
\nu_{\tau}^{I}
\end{array}\right)_{L}=V\left(\begin{array}{c}
\nu_{e} \\
\nu_{\nu} \\
\nu_{\tau}
\end{array}\right)+R\left(\begin{array}{c}
N_{e} \\
N_{\nu} \\
N_{\tau}
\end{array}\right)
$$

Unitarity of $6 \times 6$ rotation matrix leads to $V V^{\dagger}+R R^{\dagger}=I$, which implies that the MNS matrix $V$ is not unitary and the matrix $R$ characterize this non-unitarity of $V$. By testing the non-unitarity of $V$ matrix, we can examine heavy neutrino effects at low energy region 34. In the case that $h_{R} \gg h_{L}, y+y_{3}$, we can diagonalize the mass matrix approximately by

$$
\left(\begin{array}{cc}
V & R \\
S & U
\end{array}\right)=\left(\begin{array}{cc}
1-\frac{1}{8}\left(y+y_{3}\right) h_{R}^{-2}\left(y+y_{3}\right)^{T} & \frac{1}{2}\left(y+y_{3}\right) h_{R}^{-1} \\
-\frac{1}{2} h_{R}^{-1}\left(y+y_{3}\right)^{T} & 1-\frac{1}{8} h_{R}^{-1}\left(y+y_{3}\right)^{T}\left(y+y_{3}\right) h_{R}^{-1}
\end{array}\right)
$$

which will lead to

$$
\begin{align*}
\left(\begin{array}{cc}
V & R \\
S & U
\end{array}\right)^{\dagger}\left(\begin{array}{cc}
2 h_{L} & y+y_{3} \\
\left(y+y_{3}\right)^{T} & 2 h_{R}
\end{array}\right)\left(\begin{array}{cc}
V & R \\
S & U
\end{array}\right)^{*} \\
=\left(\begin{array}{cc}
2 h_{L}-\frac{1}{2}\left(y+y_{3}\right) h_{R}^{-1}\left(y+y_{3}\right)^{T}+O\left(h_{R}^{-2}\right) & O\left(h_{R}^{-1}\right) \\
O\left(h_{R}^{-1}\right) & 2 h_{R}+O\left(h_{R}^{-1}\right)
\end{array}\right) \tag{3.12}
\end{align*}
$$

If $h_{L}=0$, (3.12) leads to the standard type I seesaw mechanism, otherwise we obtain type II seesaw mechanism for neutrinos.

For quark part, (2.9) in unitary gauge is

$$
\begin{equation*}
\left.\mathcal{L}_{\mathrm{Y}, \text { quark }}\right|_{\text {Unitary gauge }}=\bar{q}_{\alpha L}^{I}\left(\tau^{u} y_{u}^{\alpha \beta}+\tau^{d} y_{d}^{\alpha \beta}\right) q_{\beta R}^{I}+\text { h.c. } \tag{3.13}
\end{equation*}
$$

We can explicitly expand coefficients $y_{u}^{\alpha \beta}, y_{d}^{\alpha \beta}$ in terms of powers of quantum fluctuation Higgs field $\tilde{h}$

$$
\begin{equation*}
y_{i}=y_{i}^{0}+y_{i}^{1} \tilde{h}+O\left(\tilde{h}^{2}\right) \quad i=u, d \tag{3.14}
\end{equation*}
$$

where $y_{i}^{0}, y_{i}^{1}$ are matrices independent of Higgs field $h$.
The gauge eigenstates can be rotated into the mass eigenstates with unitary matrices $V_{L, R}^{u, d}$,

$$
\begin{equation*}
u_{L, R}=V_{L, R}^{u} u_{L, R}^{I} \quad d_{L, R}=V_{L, R}^{d} d_{L, R}^{I} \tag{3.15}
\end{equation*}
$$

The $y_{u, d}^{0}$ matrices defined in (3.14) are diagonalized as follows:

$$
\begin{equation*}
V_{L}^{u} y_{u}^{0} V_{R}^{u \dagger}=M_{\mathrm{diag}}^{u}, \quad V_{L}^{d} y_{d}^{0} V_{R}^{d \dagger}=M_{\mathrm{diag}}^{d} \tag{3.16}
\end{equation*}
$$

where $M_{\text {diag }}^{u, d}$ represent the diagonal up- and down-quark mass matrices of physical quark masses.

$$
\begin{gather*}
q_{\alpha L, R}=\binom{u_{\alpha L, R}}{d_{\alpha L, R}}=\left[\left(V_{L, R}^{u}\right)_{\alpha \beta} \tau^{u}+\left(V_{L, R}^{d}\right)_{\alpha \beta} \tau^{d}\right]\binom{u_{\beta L, R}^{I}}{d_{\beta L, R}^{I}}  \tag{3.17}\\
\left(V_{L}^{u} \tau^{u}+V_{L}^{d} \tau^{d}\right)\left(\tau^{u} y_{u}^{0}+\tau^{d} y_{d}^{0}\right)\left(V_{R}^{u \dagger} \tau^{u}+V_{L, R}^{d \dagger} \tau^{d}\right)=\left(\tau^{u} M_{\mathrm{diag}}^{u}+\tau^{d} M_{\mathrm{diag}}^{d}\right) \tag{3.18}
\end{gather*}
$$

The usual Cabibbo-Kobayashi-Maskawa (CKM) matrix in the left sector, and the corresponding matrix in the right sector, are given by

$$
\begin{equation*}
V_{L, R}^{\mathrm{CKM}}=V_{L, R}^{u} V_{L, R}^{d \dagger} \tag{3.19}
\end{equation*}
$$

Note that, a priori, there is no reason for $V_{L}^{\mathrm{CKM}}$ to equal $V_{R}^{\mathrm{CKM}}$.
Any $n \times n$ unitary matrix has $n^{2}$ real parameters among which $n(n-1) / 2$ may be expressed in the form of $\sin \theta_{\alpha \beta}, \cos \theta_{\alpha \beta}$ with $n^{2}-n(n-1) / 2=n(n+1) / 2$ phases left. Since each quark field can be redefined through a phase transformation, $2 n-1$ phases are not physical. If $V_{L}^{\mathrm{CKM}}$ and $V_{R}^{\mathrm{CKM}}$ are independent, the total number of physical phases is
$2 \times \frac{n(n+1)}{2}-(2 n-1)=n^{2}-n+1$. In our case of 3 generations of fermions, $V_{L}^{\text {CKM }}$ can be taken as the standard form [35],

$$
V_{L}^{\mathrm{CKM}}=\left(\begin{array}{ccc}
V_{L}^{u d} & V_{L}^{u s} & V_{L}^{u b}  \tag{3.20}\\
V_{L}^{c d} & V_{L}^{c s} & V_{L}^{c b} \\
V_{L}^{t d} & V_{L}^{t s} & V_{L}^{t b}
\end{array}\right)=\left(\begin{array}{ccc}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
-s_{12} c_{23}-c_{12} s_{23} s_{13} e^{i \delta} & c_{12} c_{23}-s_{12} s_{23} s_{13} e^{i \delta} & s_{23} c_{13} \\
s_{12} s_{23}-c_{12} c_{23} s_{13} e^{i \delta} & -c_{12} s_{23}-s_{12} c_{23} s_{13} e^{i \delta} & c_{23} c_{13}
\end{array}\right)
$$

Then the most general $V_{R}^{\mathrm{CKM}}$ may be in the form of standard CKM matrix with 5 phases added:

$$
V_{R}^{\mathrm{CKM}}=\left(\begin{array}{lll}
\bar{V}_{R}^{u d} e^{2 i \alpha_{1}} & \bar{V}_{R}^{u s} e^{i\left(\alpha_{1}+\alpha_{2}+\beta_{1}\right)} & \bar{V}_{R}^{u b} e^{i\left(\alpha_{1}+\alpha_{3}+\beta_{1}+\beta_{2}\right)}  \tag{3.21}\\
\bar{V}_{R}^{c d} e^{i\left(\alpha_{1}+\alpha_{2}-\beta_{1}\right)} & \bar{V}_{R}^{c s} e^{2 i \alpha_{2}} & \bar{V}_{R}^{c b} e^{i\left(\alpha_{2}+\alpha_{3}+\beta_{2}\right)} \\
\bar{V}_{R}^{t d} e^{i\left(\alpha_{1}+\alpha_{3}-\beta_{1}-\beta_{2}\right)} & \bar{V}_{R}^{t s} e^{i\left(\alpha_{2}+\alpha_{3}-\beta_{2}\right)} & \bar{V}_{R}^{t b} e^{2 i \alpha_{3}}
\end{array}\right)
$$

where

$$
\left(\begin{array}{ccc}
\bar{V}_{R}^{u d} & \bar{V}_{R}^{u s} & \bar{V}_{R}^{u b}  \tag{3.22}\\
\bar{V}_{R}^{c d} & \bar{V}_{R}^{c s} & \bar{V}_{R}^{c b} \\
\bar{V}_{R}^{t d} & \bar{V}_{R}^{t s} & \bar{V}_{R}^{t b}
\end{array}\right)=\left(\begin{array}{ccc}
\bar{c}_{12} \bar{c}_{13} & \bar{s}_{12} \bar{c}_{13} & \bar{s}_{13} e^{-i \bar{\delta}} \\
-\bar{s}_{12} \bar{c}_{23}-\bar{c}_{12} \bar{s}_{23} \bar{s}_{13} e^{i \bar{\delta}} & \bar{c}_{12} \bar{c}_{23}-\bar{s}_{12} \bar{s}_{23} \bar{s}_{13} e^{i \bar{\delta}} & \bar{s}_{23} \bar{c}_{13} \\
\bar{s}_{12} \bar{s}_{23}-\bar{c}_{12} \bar{c}_{23} \bar{s}_{13} e^{i \bar{\delta}} & -\bar{c}_{12} \bar{s}_{23}-\bar{s}_{12} \bar{c}_{23} \bar{s}_{13} e^{i \bar{\delta}} & \bar{c}_{23} \bar{c}_{13}
\end{array}\right)
$$

with $\bar{c}_{12}=\cos \bar{\theta}_{12}, \bar{s}_{12}=\sin \bar{\theta}_{12}$, etc. In general, $\bar{\theta}_{\alpha \beta}$ do not equal to those in $V_{L}^{\text {CKM }}$. If $\bar{V}_{R}^{\alpha \beta}=\left(V_{L}^{\alpha \beta}\right)^{*}$ hold for $\alpha=u, c, t$ and $\beta=d, s, b$, then $V_{R}^{\mathrm{CKM}}$ in (3.21) coincides with that in 36, 37] which is called pseudo-manifest left-right symmetric and is originally proposed to construct left-right symmetric models with spontaneously CP violation.

## 4. Goldstone, Higgs and gauge couplings to quarks

The discussions in last section are limited in unitary gauge without the Goldstone contributions and Higgs contributions included. As a compensation and preparation of next section computation, we now focus our attention on quark-Goldstone-boson and quarkHiggs couplings. We will find that, unlike the mixing terms dealt above, these coupling are no longer universal. We explicitly expanded out Goldstone fields by

$$
U_{1,2}=\exp \left(\frac{i g_{1,2}}{\sqrt{2} M_{1,2}} \phi_{1,2}\right) \quad \phi_{1,2}=\left(\begin{array}{cc}
\frac{\phi_{1,2}^{0}}{\sqrt{2}} & \phi_{1,2}^{+}  \tag{4.1}\\
\phi_{1,2}^{-} & -\frac{\phi_{1,2}^{0}}{\sqrt{2}}
\end{array}\right) \quad M_{1,2}=\frac{1}{2} f_{1,2} g_{1,2} .
$$

in which we have taken $\zeta=0$. In terms of the masses eigenstates, Lagrangian (2.9) can be expanded according to the goldstone and Higgs fields,

$$
\begin{align*}
\mathcal{L}_{\mathrm{Y}, \text { quark }}= & \left(1+\frac{i g_{L}}{2 M_{L}} \phi_{L}^{0}-\frac{i g_{R}}{2 M_{R}} \phi_{R}^{0}\right) \bar{u}_{L} M_{\text {diag }}^{u} u_{R}+\left(1-\frac{i g_{L}}{2 M_{L}} \phi_{L}^{0}+\frac{i g_{R}}{2 M_{R}} \phi_{R}^{0}\right) \bar{d}_{L} M_{\text {diag }}^{d} d_{R} \\
& + \text { h.c. }+\mathcal{L}_{\mathrm{Y}, \mathrm{CC}}+\mathcal{L}_{\mathrm{Y}, h}+O\left(\bar{q} \phi^{2} q, \bar{q} \tilde{h}^{2} q, \bar{q} \phi \tilde{h} q\right) \tag{4.2}
\end{align*}
$$

where the charged Yukawa coupling $\mathcal{L}_{\mathrm{Y}, \mathrm{CC}}$ in Lagrangian (4.2) is

$$
\begin{align*}
\mathcal{L}_{\mathrm{Y}, \mathrm{CC}}= & -\frac{i g_{L}}{\sqrt{2} M_{L}}\left(\bar{u}_{R} M_{\mathrm{diag}}^{u} \phi_{L}^{+} V_{L}^{\mathrm{CKM}} d_{L}-\bar{u}_{L} V_{L}^{\mathrm{CKM}} \phi_{L}^{+} M_{\text {diag }}^{d} d_{R}\right) \\
& -\frac{i g_{R}}{\sqrt{2} M_{R}}\left(\bar{u}_{L} M_{\text {diag }}^{u} V_{R}^{\mathrm{CKM}} \phi_{R}^{+} d_{R}-\bar{u}_{R} \phi_{R}^{+} V_{R}^{\mathrm{CKM}} M_{\text {diag }}^{d} d_{L}\right)+\text { h.c. } \\
\equiv & -\frac{i}{\sqrt{2}} \bar{u}_{\alpha}\left(A_{L}^{\alpha \beta}+B_{L}^{\alpha \beta} \gamma^{5}\right) d_{\beta} \phi_{L}^{+}-\frac{i}{\sqrt{2}} \bar{u}_{\alpha}\left(A_{R}^{\alpha \beta}+B_{R}^{\alpha \beta} \gamma^{5}\right) d_{\beta} \phi_{R}^{+} \\
& -\frac{i}{\sqrt{2}} \bar{d}_{\beta}\left(A_{L}^{\alpha \beta \dagger}+B_{L}^{\alpha \beta \dagger} \gamma^{5}\right) u_{\alpha} \phi_{L}^{-}-\frac{i}{\sqrt{2}} \bar{d}_{\beta}\left(A_{R}^{\alpha \beta \dagger}+B_{R}^{\alpha \beta \dagger} \gamma^{5}\right) u_{\alpha} \phi_{R}^{-} \tag{4.3}
\end{align*}
$$

with

$$
\begin{align*}
A_{L}^{\alpha \beta} & =\frac{1}{2} \frac{g_{L}}{M_{L}}\left(m_{u_{\alpha}}-m_{d_{\beta}}\right) V_{L}^{\alpha \beta}, & B_{L}^{\alpha \beta} & =\frac{1}{2} \frac{g_{L}}{M_{L}}\left(-m_{u_{\alpha}}-m_{d_{\beta}}\right) V_{L}^{\alpha \beta} \\
A_{R}^{\alpha \beta} & =\frac{1}{2} \frac{g_{R}}{M_{R}}\left(m_{u_{\alpha}}-m_{d_{\beta}}\right) V_{R}^{\alpha \beta}, & B_{R}^{\alpha \beta} & =\frac{1}{2} \frac{g_{R}}{M_{R}}\left(m_{u_{\alpha}}+m_{d_{\beta}}\right) V_{R}^{\alpha \beta} \\
A_{L}^{\alpha \beta \dagger} & =\frac{1}{2} \frac{g_{L}}{M_{L}}\left(m_{d_{\beta}}-m_{u_{\alpha}}\right) V_{L}^{\alpha \beta *}, & B_{L}^{\alpha \beta \dagger} & =\frac{1}{2} \frac{g_{L}}{M_{L}}\left(-m_{d_{\beta}}-m_{u_{\alpha}}\right) V_{L}^{\alpha \beta *} \\
A_{R}^{\alpha \beta \dagger} & =\frac{1}{2} \frac{g_{R}}{M_{R}}\left(m_{d_{\beta}}-m_{u_{\alpha}}\right) V_{R}^{\alpha \beta *}, & B_{R}^{\alpha \beta \dagger} & =\frac{1}{2} \frac{g_{R}}{M_{R}}\left(m_{d_{\beta}}+m_{u_{\alpha}}\right) V_{R}^{\alpha \beta *} \tag{4.4}
\end{align*}
$$

To cover various models, we use symbol $\frac{g_{L}}{M_{L}} \phi_{L}$ and $\frac{g_{R}}{M_{R}} \phi_{R}$ to represent Goldstone fields and corresponding couplings. Their relations with Goldstone fields $\phi_{1}, \phi_{2}$ and corresponding couplings are

$$
\begin{align*}
\frac{g_{L}}{M_{L}} \phi_{L} & = \begin{cases}\frac{g_{1}}{M_{1}} \phi_{1} & \text { LR,LP,HP,FP } \\
\frac{g_{2}}{M_{2}} \phi_{2} & \text { UN } \\
\frac{g_{1}}{M_{1}} \phi_{1} \delta_{\alpha \alpha_{1}}+\frac{g_{2}}{M_{2}} \phi_{2} \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases}  \tag{4.5}\\
\frac{g_{R}}{M_{R}} \phi_{R} & = \begin{cases}\frac{g_{2}}{M_{2}} \phi_{2} & \text { LR,LP, } \\
0 & \text { HP,FP,UN,NU }\end{cases} \tag{4.6}
\end{align*}
$$

The quark-Higgs-boson couplings $\mathcal{L}_{Y, h}$ in Lagrangian (4.2) are

$$
\begin{align*}
\mathcal{L}_{\mathrm{Y}, h} & =\tilde{h}\left(\bar{u}_{L} V_{L}^{u} y_{u}^{1} V_{R}^{u \dagger} u_{R}+\bar{d}_{L} V_{L}^{d} y_{d}^{1} V_{R}^{d \dagger} d_{R}\right)+\text { h.c. } \\
& =\frac{1}{2} \bar{u}_{\alpha}\left(A_{u}^{\alpha \beta}+B_{u}^{\alpha \beta} \gamma^{5}\right) u_{\beta} \tilde{h}+\frac{1}{2} \bar{d}_{\alpha}\left(A_{d}^{\alpha \beta}+B_{d}^{\alpha \beta} \gamma^{5}\right) d_{j} \beta \tilde{h} \tag{4.7}
\end{align*}
$$

where

$$
\begin{array}{ll}
A_{u}^{\alpha \beta}=\left(\tilde{y}_{u}+\tilde{y}_{u}^{\dagger}\right)^{\alpha \beta}, & B_{u}^{\alpha \beta}=\left(\tilde{y}_{u}-\tilde{y}_{u}^{\dagger}\right)^{\alpha \beta}, \\
A_{d}^{\alpha \beta}=\left(\tilde{y}_{d}+\tilde{y}_{d}^{\dagger}\right)^{\alpha \beta}, & B_{d}^{\alpha \beta}=\left(\tilde{y}_{d}-\tilde{y}_{d}^{\dagger}\right)^{\alpha \beta}, \tag{4.8}
\end{array}
$$

where $\tilde{y}_{u}=V_{L}^{u} y_{u}^{1} V_{R}^{u \dagger}$ and $\tilde{y}_{d}=V_{L}^{d} y_{d}^{1} V_{R}^{d \dagger}$. Note that for neutral goldstones, there is no flavor-changing $\bar{q} \phi q$ coupling. For charged goldstone bosons, the non-diagonal CKM matrices and nontrivial mass difference of quarks will yields flavor-changing couplings. If $y_{i}^{1}=y_{i}^{0} / v$ with $v$ the expectation value of $h$, the quark-Higgs couplings is in agree with
that of the SM. However, flavor-changing couplings for neutral Higgs field $\tilde{h}$ can exist in general due to the fact that matrices $V_{L}^{i} y_{i}^{1} V_{R}^{i \dagger}$ may not be diagonal.

Now we come to discuss gauge couplings. In unitary gauge, Lagrangian (2.10) become

$$
\begin{align*}
\left.\mathcal{L}_{f-4}\right|_{\text {Unitary gauge }}= & \sum_{\alpha} \bar{q}_{\alpha}^{I}\left[i \not \partial \boldsymbol{\partial}-\left(\Delta_{1,1, \alpha} g_{1} \frac{\tau^{a^{\prime}}}{2} W_{1}^{a^{\prime}}+\Delta_{1,2, \alpha} \frac{\tau^{a^{\prime}}}{2} g_{2} W_{2}^{a^{\prime}}+\Delta_{1,1, \alpha}^{3} g_{1} W_{1}^{3}\right.\right.  \tag{4.9}\\
& \left.+\Delta_{1,2, \alpha}^{3} g_{2} W_{2}^{3}+\Delta_{1, \alpha} g \not B\right) P_{L}-\left(\Delta_{2,1, \alpha} g_{1} \frac{\tau^{a^{\prime}}}{2} W_{1}^{a^{\prime}}+\Delta_{2,2, \alpha} \frac{\tau^{a^{\prime}}}{2} g_{2} W_{2}^{a^{\prime}}\right. \\
& \left.\left.+\Delta_{2,1, \alpha}^{3} g_{1} W_{1}^{3}+\Delta_{2,2, \alpha}^{3} g_{2} \mathscr{W}_{2}^{3}+\Delta_{2, \alpha} g \not B\right) P_{R}\right] q_{\alpha}^{I}+q^{I} \rightarrow l^{I}, \delta \rightarrow \delta^{l}, \Delta \rightarrow \Delta^{l},
\end{align*}
$$

where $a^{\prime}=1,2$. The above anomalous gauge couplings $\Delta$ 's can be expressed by $\delta$ 's introduced in (2.10) and the detailed results are given in appendix A. In terms of mass eigenstates for gauge bosons, (4.9) become

$$
\begin{equation*}
\left.\mathcal{L}_{f-4}\right|_{\text {Unitary gauge }}=i \bar{q}_{\alpha}^{I} \gamma^{\mu} \partial_{\mu} q_{\alpha}^{I}+\mathcal{L}_{\mathrm{CC}}+\mathcal{L}_{\mathrm{NC}}+\mathcal{L}_{\mathrm{EM}} \tag{4.10}
\end{equation*}
$$

with charge current part $\mathcal{L}_{\mathrm{CC}}$

$$
\begin{align*}
\mathcal{L}_{\mathrm{CC}}= & \frac{-1}{\sqrt{2}} \sum_{\alpha}\left[\bar{q}_{\alpha}^{I}\left[\left(g_{1} \cos \zeta \Delta_{1,1, \alpha}+g_{2} \sin \zeta \Delta_{1,2, \alpha}\right) \gamma^{\mu} P_{L}+\left(g_{2} \sin \zeta \Delta_{2,1, \alpha}+g_{1} \cos \zeta \Delta_{2,2, \alpha}\right) \gamma^{\mu} P_{R}\right]\right. \\
& \times\left(\tau^{+} W_{\mu}^{+}+\tau^{-} W_{\mu}^{-}\right) q_{\alpha}^{I}+\bar{q}_{\alpha}^{I}\left[\left(-g_{1} \sin \zeta \Delta_{1,1, \alpha}+g_{2} \cos \zeta \Delta_{1,2, \alpha}\right) \gamma^{\mu} P_{L}+\left(g_{2} \cos \zeta \Delta_{2,1, \alpha}\right.\right. \\
& \left.\left.\left.-g_{1} \sin \zeta \Delta_{2,2, \alpha}\right) \gamma^{\mu} P_{R}\right]\left(\tau^{+} W_{\mu}^{\prime+}+\tau^{-} W_{\mu}^{\prime-}\right) q_{\alpha}^{I}\right]+q^{I} \rightarrow l^{I}, \Delta \rightarrow \Delta^{l}, \tag{4.11}
\end{align*}
$$

neutral current part $\mathcal{L}_{\mathrm{NC}}$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{NC}}= & -\frac{1}{2} \sum_{\alpha}\left[\overline { q } _ { \alpha } ^ { I } \left\{\left[g_{1} x_{1}\left(\Delta_{1,1, \alpha}^{3}+\Delta_{2,2, \alpha}^{3}\right)+g_{2} y_{1}\left(\Delta_{2,1, \alpha}^{3}+\Delta_{1,2, \alpha}^{3}\right)+g v_{1}\left(\Delta_{1, \alpha}+\Delta_{2, \alpha}\right)\right] \gamma^{\mu}\right.\right. \\
& \left.-\left[g_{1} x_{1}\left(\Delta_{1,1, \alpha}^{3}-\Delta_{2,2, \alpha}^{3}\right)-g_{2} y_{1}\left(\Delta_{2,1, \alpha}^{3}-\Delta_{1,2, \alpha}^{3}\right)+g v_{1}\left(\Delta_{1, \alpha}-\Delta_{2, \alpha}\right)\right] \gamma^{\mu} \gamma^{5}\right\} q_{\alpha}^{I} Z_{\mu} \\
& +\bar{q}_{\alpha}^{I}\left\{\left[g_{1} x_{2}\left(\Delta_{1,1, \alpha}^{3}+\Delta_{2,2, \alpha}^{3}\right)+g_{2} y_{2}\left(\Delta_{2,1, \alpha}^{3}+\Delta_{1,2, \alpha}^{3}\right)+g v_{2}\left(\Delta_{1, \alpha}+\Delta_{2, \alpha}\right)\right] \gamma^{\mu}\right. \\
& \left.\left.-\left[g_{1} x_{2}\left(\Delta_{1,1, \alpha}^{3}-\Delta_{2,2, \alpha}^{3}\right)-g_{1} y_{2}\left(\Delta_{2,1, \alpha}^{3}-\Delta_{1,2, \alpha}^{3}\right)+g v_{2}\left(\Delta_{1, \alpha}-\Delta_{2, \alpha}\right)\right] \gamma^{\mu} \gamma^{5}\right\} q_{\alpha}^{I} Z_{\mu}^{\prime}\right] \\
& +q^{I} \rightarrow l^{I}, \Delta \rightarrow \Delta^{l},
\end{aligned}
$$

electro-magnetic current part $\mathcal{L}_{\text {EM }}$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{EM}}= & -\frac{1}{2} \sum_{\alpha} \bar{q}_{\alpha}^{I}\left\{\left[g_{1} x_{3}\left(\Delta_{1,1, \alpha}^{3}+\Delta_{2,2, \alpha}^{3}\right)+g_{2} y_{3}\left(\Delta_{2,1, \alpha}^{3}+\Delta_{1,2, \alpha}^{3}\right)+g v_{3}\left(\Delta_{1, \alpha}+\Delta_{2, \alpha}\right)\right] \gamma^{\mu}\right. \\
& \left.-\left[g_{1} x_{3}\left(\Delta_{1,1, \alpha}^{3}-\Delta_{2,2, \alpha}^{3}\right)-g_{2} y_{3}\left(\Delta_{2,1, \alpha}^{3}-\Delta_{1,2, \alpha}^{3}\right)+g v_{3}\left(\Delta_{1, \alpha}-\Delta_{2, \alpha}\right)\right] \gamma^{\mu} \gamma^{5}\right\} q_{\alpha}^{I} A_{\mu} \\
& +q^{I} \rightarrow l^{I}, \Delta \rightarrow \Delta^{l},
\end{aligned}
$$

Further in terms of fermion mass eigenstates, $\mathcal{L}_{\mathrm{NC}}$ and $\mathcal{L}_{\mathrm{EM}}$ keep their present form, but we must replace original summation over generation indices $\sum_{\alpha} \bar{q}_{\alpha}^{I} \Delta_{i, \alpha} q_{\alpha}^{I}$ with $\sum_{\alpha \beta} \bar{q}_{\alpha} \Delta_{i, \alpha \beta}^{\prime} q_{\beta}$
where

$$
\begin{align*}
\Delta_{i, \alpha \beta}^{\prime} \equiv & {\left[V_{L}^{u} \operatorname{diag}\left(\Delta_{i, 1}, \Delta_{i, 2}, \Delta_{i, 3}\right) V_{L}^{u \dagger}+V_{R}^{u} \operatorname{diag}\left(\Delta_{i, 1}, \Delta_{i, 2}, \Delta_{i, 3}\right) V_{R}^{u \dagger}\right]_{\alpha \beta} \tau^{u} } \\
& +\left[V_{L}^{d} \operatorname{diag}\left(\Delta_{i, 1}, \Delta_{i, 2}, \Delta_{i, 3}\right) V_{L}^{d \dagger}+V_{R}^{d} \operatorname{diag}\left(\Delta_{i, 1}, \Delta_{i, 2}, \Delta_{i, 3}\right) V_{R}^{d \dagger}\right]_{\alpha \beta} \tau^{d} \tag{4.12}
\end{align*}
$$

It is easy to see that if $\Delta_{i, \alpha}$ is universal in generation, i.e. it is independent of index $\alpha$, then $\Delta_{i, \alpha \beta}^{\prime}=\Delta_{i, \alpha} \delta_{\alpha \beta}$ which leads $\mathcal{L}_{\mathrm{NC}}$ and $\mathcal{L}_{\text {EM }}$ unchanged. In order to suppress the possible flavor changing neutral and electro-magnetic currents, either non-universal effect of $\Delta_{i, \alpha}$ appeared in $\mathcal{L}_{\mathrm{NC}}$ and $\mathcal{L}_{\mathrm{EM}}$ is small or there is some cancelations among different terms in (4.12).

The charge current Lagrangian for quarks in mass eigenstates is changed to

$$
\begin{equation*}
\mathcal{L}_{\mathrm{CC}}=-\frac{1}{\sqrt{2}} \bar{u}_{\alpha} \gamma^{\mu}\left(A_{W}^{\alpha \beta}+B_{W}^{\alpha \beta} \gamma^{5}\right) d_{\beta} W_{\mu}^{+}-\frac{1}{\sqrt{2}} \bar{u}_{\alpha} \gamma^{\mu}\left(A_{W^{\prime}}^{\alpha \beta}+B_{W^{\prime}}^{\alpha \beta} \gamma^{5}\right) d_{\beta} W_{\mu}^{\prime+}+\text { h.c. } \tag{4.13}
\end{equation*}
$$

where

$$
\begin{align*}
A_{W}^{\alpha \beta}=\frac{1}{2}[ & g_{1} \cos \zeta\left[V_{L}^{u} \operatorname{diag}\left(\Delta_{1,1,1}, \Delta_{1,1,2}, \Delta_{1,1,3}\right) V_{L}^{d \dagger}+V_{R}^{u} \operatorname{diag}\left(\Delta_{2,2,1}, \Delta_{2,2,2}, \Delta_{2,2,3}\right) V_{R}^{d \dagger}\right]_{\alpha \beta} \\
& \left.+g_{2} \sin \zeta\left[V_{L}^{u} \operatorname{diag}\left(\Delta_{1,2,1}, \Delta_{1,2,2}, \Delta_{1,2,3}\right) V_{L}^{d \dagger}+V_{R}^{u} \operatorname{diag}\left(\Delta_{2,1,1}, \Delta_{2,1,2}, \Delta_{2,1,3}\right) V_{R}^{d \dagger}\right]_{\alpha \beta}\right] \\
B_{W}^{\alpha \beta}=\frac{1}{2} & {\left[g_{1} \cos \zeta\left[-V_{L}^{u} \operatorname{diag}\left(\Delta_{1,1,1}, \Delta_{1,1,2}, \Delta_{1,1,3}\right) V_{L}^{d \dagger}+V_{R}^{u} \operatorname{diag}\left(\Delta_{2,2,1}, \Delta_{2,2,2}, \Delta_{2,2,3}\right) V_{R}^{d \dagger}\right]_{\alpha \beta}\right.} \\
& \left.+g_{2} \sin \zeta\left[-V_{L}^{u} \operatorname{diag}\left(\Delta_{1,2,1}, \Delta_{1,2,2}, \Delta_{1,2,3}\right) V_{L}^{d \dagger}+V_{R}^{u} \operatorname{diag}\left(\Delta_{2,1,1}, \Delta_{2,1,2}, \Delta_{2,1,3}\right) V_{R}^{d \dagger}\right]_{\alpha \beta}\right] \\
A_{W^{\prime}}^{\alpha \beta}=\frac{1}{2}[ & g_{2} \cos \zeta\left[V_{R}^{u} \operatorname{diag}\left(\Delta_{2,1,1}, \Delta_{2,1,2}, \Delta_{2,1,3}\right) V_{R}^{d \dagger}+V_{L}^{u} \operatorname{diag}\left(\Delta_{1,2,1}, \Delta_{1,2,2}, \Delta_{1,2,3}\right) V_{L}^{d \dagger}\right]_{\alpha \beta} \\
& \left.-g_{1} \sin \zeta\left[V_{R}^{u} \operatorname{diag}\left(\Delta_{2,2,1}, \Delta_{2,2,2}, \Delta_{2,2,3}\right) V_{R}^{d \dagger}+V_{L}^{u} \operatorname{diag}\left(\Delta_{1,1,1}, \Delta_{1,1,2}, \Delta_{1,1,3}\right) V_{L}^{d \dagger}\right]_{\alpha \beta}\right] \\
B_{W^{\prime}}^{\alpha \beta}=\frac{1}{2}[ & -g_{2} \cos \zeta\left[-V_{R}^{u} \operatorname{diag}\left(\Delta_{2,1,1}, \Delta_{2,1,2}, \Delta_{2,1,3}\right) V_{R}^{d \dagger}+V_{L}^{u} \operatorname{diag}\left(\Delta_{1,2,1}, \Delta_{1,2,2}, \Delta_{1,2,3}\right) V_{L}^{d \dagger}\right]_{\alpha \beta} \\
& \left.+g_{1} \sin \zeta\left[-V_{R}^{u} \operatorname{diag}\left(\Delta_{2,2,1}, \Delta_{2,2,2}, \Delta_{2,2,3}\right) V_{R}^{d \dagger}+V_{L}^{u} \operatorname{diag}\left(\Delta_{1,1,1}, \Delta_{1,1,2}, \Delta_{1,1,3}\right) V_{L}^{d \dagger}\right]_{\alpha \beta}\right] \tag{4.14}
\end{align*}
$$

If $\Delta_{i, \alpha}$ is universal in generation index, then rotation matrices appeared in above formulae will meet together constituting CKM matrices.

If we only focus on gauge couplings to light gauge boson $A, W, Z$, above $\Delta$ 's cause a serious anomalous couplings. In ref. [28], we parameterized these anomalous couplings in terms of ten coefficients in the case that $\Delta_{i, \alpha}$ is universal in generation index, for which two are in charged current, four in neutral current and four in electro-magnetic current. The fact that SM is consistent with experiment to very high precision implies these ten anomalous couplings must be very small in values.

## 5. Effective Hamiltonian for neutral $K$ and $B$ system

Once there exists $W^{\prime}$ boson, there may be low energy phenomenological constraints from $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}$ and $B_{s}^{0}-\bar{B}_{s}^{0}$ system. In most cases $W^{\prime}$ will generate extra Feynman


Figure 1: Box diagrams for $K^{0}-\bar{K}^{0}$ effective Hamiltonian $H_{\text {eff }}^{\mathrm{WW}}$.


Figure 2: Higgs exchange diagrams for $K^{0}-\bar{K}^{0}$ effective Hamiltonian.
box diagrams which contribute to mass differences in $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}, B_{s}^{0}-\bar{B}_{s}^{0}$ system and corresponding CP violation parameters. These mixings are described by a effective Hamiltonian which is composed of four parts:

$$
\begin{equation*}
H_{\mathrm{eff}}=H_{\mathrm{eff}}^{\mathrm{WW}}+H_{\mathrm{eff}}^{W^{\prime} W^{\prime}}+H_{\mathrm{eff}}^{W W^{\prime}}+H_{\mathrm{eff}}^{h^{0}} \tag{5.1}
\end{equation*}
$$

The effective Hamilton $H_{\text {eff }}$ is model dependent. We take the $K^{0}-\bar{K}^{0}$ system in LR and LP models as an example, other models and $B^{0}-\bar{B}^{0}$ system can be given in the similar way. The $W W$ box diagram for $K^{0}-\bar{K}^{0}$ system is plotted in figure 1. Let $M^{\mathrm{XY}}$ be the amplitudes of the diagram mediated by particles $X$ and $Y$ which may be gauge bosons $W, W^{\prime}$ and goldstone bosons $\phi_{1}, \phi_{2} . H_{\text {eff }}^{W W}$ can be further decomposed into $H_{\mathrm{eff}}^{\mathrm{WW}}=\frac{1}{2}\left(M^{\mathrm{WW}}+M^{W \phi_{1}}+M^{\phi_{1} W}+M^{\phi_{1} \phi_{1}}\right)+$ h.c.
$W^{\prime} W^{\prime}$ box diagram part $H_{\mathrm{eff}}^{W^{\prime} W^{\prime}}$ can be obtained from $H_{\mathrm{eff}}^{\mathrm{WW}}$ by $H_{\mathrm{eff}}^{W^{\prime} W^{\prime}}=$ $\left.H_{\text {eff }}^{\mathrm{WW}}\right|_{W \rightarrow W^{\prime}, 1 \rightarrow 2}$. Similarly $W W^{\prime}$ box diagram part $H_{\mathrm{eff}}^{W W^{\prime}}$ is $H_{\mathrm{eff}}^{W W^{\prime}} \stackrel{\text { ef }}{=} M^{W W^{\prime}}+M^{W \phi_{2}}+$ $M^{\phi_{1} W^{\prime}}+M^{\phi_{1} \phi_{2}}+$ h.c.. $H_{\mathrm{eff}}^{h^{0}}$ is the part of effective Hamiltonian arises from the flavor changing Yukawa coupling via neutral Higgs exchange at tree level. The corresponding Feynman diagrams is plot in figure 2 for $K^{0}-\bar{K}^{0}$ system.

The $W^{\prime}$ dependent part of $H_{\text {eff }}$ introduced in (5.1) is also model dependent. LR and LP models are main cases we are going to discuss in which $H_{\text {eff }}^{W^{\prime} W^{\prime}}$ is usually neglected due to existence of a suppression factor $\left(M_{W} / M_{W^{\prime}}\right)^{4}$. SM calculation shows that just SM effect in $H_{\text {eff }}^{\mathrm{WW}}$ itself can already match experiment data. Therefore the constraints left is that either non SM effects in $H_{\mathrm{eff}}^{\mathrm{WW}}, H_{\mathrm{eff}}^{W W^{\prime}}$ and $H_{\text {eff }}^{h^{0}}$ are all small in values separately or they cancel each other. The cancelation will demand detailed model arrangements which need fine tuning such as introducing in theory the second bi-doublet higgs discussed in
ref. [37. In this work we do not consider this special fine tuning situation and only limit us in the case that all non SM effects in $H_{\mathrm{eff}}^{\mathrm{WW}}, H_{\mathrm{eff}}^{W W^{\prime}}$ and $H_{\text {eff }}^{h^{0}}$ are small separately in values. This choice is in accordance with the approximation that only dimension three and four matter part operators are included in our calculation. If we consider more higher dimension operators, dimension six four quark operators such as $\bar{d}_{R} s_{L} \bar{d}_{L} s_{R}$ will contribute to $H_{\text {eff }}$ as a contact term. This will raise the possibility that using four quark operator contributions to cancel non SM effects in $H_{\mathrm{eff}}^{\mathrm{WW}}, H_{\mathrm{eff}}^{W W^{\prime}}$ and $H_{\text {eff }}^{h^{0}}$. This four quark operator can be seen as remnant of exchanging some more heavier unknown particles and the coupling of the operator is proportional to inverse of heavy particle mass square, like traditional Fermi weak interaction theory induced by exchanging electroweak gauge bosons. In our treatment we have ignored possible cancelations among operators of different classes. If we generalize this treatment to higher dimension operators, the cancelations among contributions of four quark operators and $W, W^{\prime}, h^{0}$ to $H_{\text {eff }}$ are not allowed. This implies the effective coupling in front of corresponding four quark operator must be small which will improve the convergence of our expansion and we can safely drop out four quark operator in our first order approximation. This is the discussion for LR and LP models. The situation in NU model is similar as in LR and LP models, except there exists explicit non-universality term in (4.14). For other models, HP and FP models are irrelevant, since in these models $W^{\prime}$ does not couple to light ordinary quarks if we ignore small mixing between $W$ and $W^{\prime}$. Then there are approximately no $H_{\mathrm{eff}}^{W^{\prime} W^{\prime}}$ and $H_{\mathrm{eff}}^{W W^{\prime}}$ terms in (5.1). The only constraint for these models is the value of $H_{\text {eff }}^{h^{0}}$ must be small. In the case of UN model, $W$ does not couple to ordinary quark if we ignore mixing between $W$ and $W^{\prime}$. The role of $W$ is replaced by $W^{\prime}$. Considering the facts that $H_{\text {eff }}^{W^{\prime} W^{\prime}}$ is much smaller than $H_{\mathrm{eff}}^{\mathrm{WW}}$ in value due to suppression factor and there is no $H_{\mathrm{eff}}^{\mathrm{WW}}$ and $H_{\mathrm{eff}}^{W W^{\prime}}$ terms, the value of $H_{\text {eff }}^{h^{0}}$ in this case can be larger than that of HP and FP models. Since the constraints for UN model from mass differences in $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}, B_{s}^{0}-\bar{B}_{s}^{0}$ system and corresponding CP violation parameters are relatively weak, we skip the discussion of this situation. Combining above discussions together, for the $W^{\prime}$ dependent part of $H_{\text {eff }}$, we only need to discuss two situations: one is LR and LP models, the other is NU model.

In performing detailed computations for box diagrams, we choose Feynman gauge and take the masses and four-momenta of the external legs to be zero ( $m_{d}=m_{s}=0$ ) thus the internal lines carry the same momentum. Detailed calculations give following amplitudes for the diagram mediated by $X-Y, X, Y=W, W^{\prime}$,

$$
\begin{aligned}
& M^{\mathrm{XY}}=\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta} \sqrt{x_{\alpha} x_{\beta}}\left[4 I_{1}\left(x_{\alpha}, x_{\beta}, \beta\right)\right] \bar{d}\left[\left(A_{X}^{\alpha s} A_{Y}^{\alpha d \dagger}-B_{X}^{\alpha s} B_{Y}^{\alpha d \dagger}\right)\right. \\
& \left.+\gamma_{5}\left(B_{X}^{\alpha s} A_{Y}^{\alpha d \dagger}-A_{X}^{\alpha s} B_{Y}^{\alpha d \dagger}\right)\right] s \otimes \bar{d}\left[\left(A_{Y}^{\beta s} A_{X}^{\beta d \dagger}-B_{Y}^{\beta s} B_{X}^{\beta d \dagger}\right)+\gamma_{5}\left(B_{Y}^{\beta s} A_{X}^{\beta d \dagger}-A_{Y}^{\beta s} B_{X}^{\beta d \dagger}\right)\right] s \\
& +\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta}\left\{[ \frac { 1 } { 4 } I _ { 2 } ( x _ { \alpha } , x _ { \beta } , \beta ) ] \left[1 0 \overline { d } \gamma _ { \mu } \left[\left(A_{X}^{\alpha s} A_{Y}^{\alpha d \dagger}+B_{X}^{\alpha s} B_{Y}^{\alpha d \dagger}\right)\right.\right.\right. \\
& \left.+\gamma_{5}\left(A_{X}^{\alpha s} B_{Y}^{\alpha d \dagger}+B_{X}^{\alpha s} A_{Y}^{\alpha d \dagger}\right)\right] s \otimes \bar{d} \gamma^{\mu}\left[\left(A_{Y}^{\beta s} A_{X}^{\beta d \dagger}+B_{Y}^{\beta s} B_{X}^{\beta d \dagger}\right)+\gamma_{5}\left(A_{Y}^{\beta s} B_{X}^{\beta d \dagger}+v B_{Y}^{\beta s} A_{X}^{\beta d \dagger}\right)\right] s
\end{aligned}
$$

$$
\begin{align*}
& -6 \bar{d} \gamma_{\mu}\left[\gamma_{5}\left(A_{X}^{\alpha s} A_{Y}^{\alpha d \dagger}+B_{X}^{\alpha s} B_{Y}^{\alpha d \dagger}\right)+\left(A_{X}^{\alpha s} B_{Y}^{\alpha d \dagger}+B_{X}^{\alpha s} A_{Y}^{\alpha d \dagger}\right)\right] s \\
& \left.\left.\otimes \bar{d} \gamma^{\mu}\left[\gamma_{5}\left(A_{Y}^{\beta s} A_{X}^{\beta d \dagger}+B_{Y}^{\beta s} B_{X}^{\beta d \dagger}\right)+\left(A_{Y}^{\beta s} B_{X}^{\beta d \dagger}+B_{Y}^{\beta s} A_{X}^{\beta d \dagger}\right)\right]\right] s\right\} \tag{5.2}
\end{align*}
$$

where $x_{\alpha}=m_{\alpha}^{2} / M_{X}^{2}$ and $\beta=M_{X}^{2} / M_{Y}^{2}$, and $A_{W}^{\alpha \beta}, A_{W^{\prime}}^{\alpha \beta}, B_{W}^{\alpha \beta}, B_{W^{\prime}}^{\alpha \beta}$ are defined in (4.14),

$$
\begin{align*}
& I_{1}\left(x_{\alpha}, x_{\beta}, \beta\right)=\frac{x_{\alpha} \ln x_{\alpha}}{\left(1-x_{\alpha}\right)\left(1-x_{\alpha} \beta\right)\left(x_{\alpha}-x_{\beta}\right)}+(\alpha \leftrightarrow \beta)-\frac{\beta \ln \beta}{(1-\beta)\left(1-x_{\alpha} \beta\right)\left(1-x_{\beta} \beta\right)} \\
& I_{2}\left(x_{\alpha}, x_{\beta}, \beta\right)=\frac{x_{\alpha}^{2} \ln x_{\alpha}}{\left(1-x_{\alpha}\right)\left(1-x_{\alpha} \beta\right)\left(x_{\alpha}-x_{\beta}\right)}+(\alpha \leftrightarrow \beta)-\frac{\ln \beta}{(1-\beta)\left(1-x_{\alpha} \beta\right)\left(1-x_{\beta} \beta\right)} \tag{5.3}
\end{align*}
$$

The amplitude of the diagram mediated by $X^{+}-\phi_{n}^{-}, X=W, W^{\prime}, n=1,2$ is:

$$
\begin{align*}
& M^{X \phi_{n}}=\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta} \sqrt{x_{\alpha} x_{\beta}}\left[I_{1}\left(x_{\alpha}, x_{\beta}, \beta\right)\right] \bar{d} \gamma_{\nu}\left[\left(A_{X}^{\alpha s} A_{n}^{\alpha d \dagger}-B_{X}^{\alpha s} B_{n}^{\alpha d \dagger}\right)\right.  \tag{5.4}\\
& \left.+\gamma_{5}\left(B_{X}^{\alpha s} A_{n}^{\alpha d \dagger}-A_{X}^{\alpha s} B_{n}^{\alpha d \dagger}\right)\right] s \otimes \bar{d} \gamma^{\nu}\left[\left(A_{n}^{\beta s} A_{X}^{\beta d \dagger}+B_{n}^{\beta s} B_{X}^{\beta d \dagger}\right)+\gamma_{5}\left(B_{n}^{\beta s} A_{X}^{\beta d \dagger}+A_{n}^{\beta s} B_{X}^{\beta d \dagger}\right)\right] s \\
& +\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta}\left[I_{2}\left(x_{\alpha}, x_{\beta}, \beta\right)\right] \bar{d}\left[\left(A_{X}^{\alpha s} A_{n}^{\alpha d \dagger}+B_{X}^{\alpha s} B_{n}^{\alpha d \dagger}\right)\right. \\
& \left.+\gamma_{5}\left(A_{X}^{\alpha s} B_{n}^{\alpha d \dagger}+B_{X}^{\alpha s} A_{n}^{\alpha d \dagger}\right)\right] s \otimes \bar{d}\left[\left(A_{n}^{\beta s} A_{X}^{\beta d \dagger}-B_{n}^{\beta s} B_{X}^{\beta d \dagger}\right)+\gamma_{5}\left(-A_{n}^{\beta s} B_{X}^{\beta d \dagger}+B_{n}^{\beta s} A_{X}^{\beta d \dagger}\right)\right] s
\end{align*}
$$

The amplitude of the diagram mediated by $\phi_{m}^{+}-Y^{-}, m=1,2, Y=W, W^{\prime}$ is,

$$
\begin{align*}
& M^{\phi_{m} Y}=\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta} \sqrt{x_{\alpha} x_{\beta}}\left[I_{1}\left(x_{\alpha}, x_{\beta}, \beta\right)\right] \bar{d} \gamma_{\mu}\left[\left(A_{m}^{\alpha s} A_{Y}^{\alpha d \dagger}+B_{m}^{\alpha s} B_{Y}^{\alpha d \dagger}\right)\right.  \tag{5.5}\\
& \left.+\gamma_{5}\left(B_{m}^{\alpha s} A_{Y}^{\alpha d \dagger}+A_{m}^{\alpha s} B_{Y}^{\alpha d \dagger}\right)\right] s \otimes \bar{d}\left[\gamma^{\mu}\left(A_{Y}^{\beta s} A_{m}^{\beta d \dagger}-B_{Y}^{\beta s} B_{m}^{\beta d \dagger}\right)+\gamma_{5}\left(B_{Y}^{\beta s} A_{m}^{\beta d \dagger}-A_{Y}^{\beta s} B_{m}^{\beta d \dagger}\right)\right] s \\
& +\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta}\left\{[ \frac { 1 } { 4 } I _ { 2 } ( x _ { \alpha } , x _ { \beta } , \beta ) ] \overline { d } \left[\left(A_{m}^{\alpha s} A_{Y}^{\alpha d \dagger}-B_{m}^{\alpha s} B_{Y}^{\alpha d \dagger}\right)\right.\right. \\
& \left.\left.+\gamma_{5}\left(-A_{m}^{\alpha s} B_{Y}^{\alpha d \dagger}+B_{m}^{\alpha s} A_{Y}^{\alpha d \dagger}\right)\right] s \otimes \bar{d}\left[\left(A_{Y}^{\beta s} A_{m}^{\beta d \dagger}+B_{Y}^{\beta s} B_{m}^{\beta d \dagger}\right)+\gamma_{5}\left(A_{Y}^{\beta s} B_{m}^{\beta d \dagger}+B_{Y}^{\beta s} A_{m}^{\beta d \dagger}\right)\right] s\right\}
\end{align*}
$$

The amplitude of the diagram mediated by $\phi_{m}^{+}-\phi_{n}^{-}, m, n=1,2$ is,

$$
\begin{align*}
& M^{\phi_{m} \phi_{n}}=\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta} \sqrt{x_{\alpha} x_{\beta}}\left[I_{1}\left(x_{\alpha}, x_{\beta}, \beta\right)\right] \bar{d}\left[\left(A_{m}^{\alpha s} A_{n}^{\alpha d \dagger}+B_{m}^{\alpha s}\right) B_{n}^{\alpha d \dagger}\right.  \tag{5.6}\\
& \left.+\gamma_{5}\left(B_{m}^{\alpha s} A_{n}^{\alpha d \dagger}+A_{m}^{\alpha s} B_{n}^{\alpha d \dagger}\right)\right] s \otimes \bar{d}\left[\left(A_{n}^{\beta s} A_{m}^{\beta d \dagger}+B_{n}^{\beta s} B_{m}^{\beta d \dagger}\right)+\gamma_{5}\left(B_{n}^{\beta s} A_{m}^{\beta d \dagger}+A_{n}^{\beta s} B_{m}^{\beta d \dagger}\right)\right] s \\
& +\left(\frac{\sqrt{2} g_{1}^{2}}{8 M_{X}^{2}}\right)^{2} \frac{M_{X}^{2}}{2 \pi^{2} g_{1}^{4}} \beta \sum_{\alpha, \beta}\left\{[ \frac { 1 } { 4 } I _ { 2 } ( x _ { \alpha } , x _ { \beta } , \beta ) ] \overline { d } \gamma _ { \rho } \left[\left(A_{m}^{\alpha s} A_{n}^{\alpha d \dagger}-B_{m}^{\alpha s} B_{n}^{\alpha d \dagger}\right)+\gamma_{5}\left(-A_{m}^{\alpha s} B_{n}^{\alpha d \dagger}\right.\right.\right. \\
& \left.\left.\left.+B_{m}^{\alpha s} A_{n}^{\alpha d \dagger}\right)\right] s \otimes \bar{d} \gamma^{\rho}\left[\left(A_{n}^{\beta s} A_{m}^{\beta d \dagger}-B_{n}^{\beta s} B_{m}^{\beta d \dagger}\right)+\gamma_{5}\left(-A_{n}^{\beta s} B_{m}^{\beta d \dagger}+B_{n}^{\beta s} A_{m}^{\beta d \dagger}\right)\right] s\right\}
\end{align*}
$$

Since the mixing angle $\zeta$ is expected to be small, for simplicity in the following we take $\zeta=0$.

For LR and LP models, we can ignore the generation index $\alpha$ dependence in all $\Delta$ 's appeared in (4.14), then the rotation matrices $V^{u}$ and $V^{d \dagger}$ can meet together forming CKM matrices. We introduce CKM factors $\lambda_{\alpha}^{\mathrm{LR}}(K)=V_{L}^{\mathrm{CKM}, u_{\alpha} s} V_{R}^{\mathrm{CKM}, u_{\alpha} d *}$ for $K^{0}-\bar{K}^{0}$ system, $\lambda_{\alpha}^{\mathrm{LR}}\left(B_{q}\right)=V_{L}^{\mathrm{CKM}, u_{\alpha} b} V_{R}^{\mathrm{CKM}, u_{\alpha} q^{*}}$ for $B_{q}^{0}-\bar{B}_{q}^{0}$ system, etc. By taking $m_{u}=0$ and using the relation $\lambda_{u}+\lambda_{c}+\lambda_{t}=0$, ignoring the higher order of $\Delta_{2,2}, \Delta_{1,2}$ (we have dropped out their generation indices) and accurate to the order linear in $\beta=M_{W}^{2} / M_{W^{\prime}}^{2}$, we obtain

$$
\begin{align*}
& H_{\mathrm{eff}}^{\mathrm{WW}}=\frac{G_{F}^{2} M_{W}^{2}}{16 \pi^{2}} \times\left\{\begin{array}{ll}
f_{\mathrm{LL}}(K) \bar{d} \gamma^{\mu}\left(1-\gamma_{5}\right) s \otimes \bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) s & K^{0}-\bar{K}^{0} \text { system } \\
f_{\mathrm{LL}}\left(B_{q}\right) \bar{q} \gamma^{\mu}\left(1-\gamma_{5}\right) b \otimes \bar{q} \gamma_{\mu}\left(1+\gamma_{5}\right) b & B_{q}^{0}-B_{\bar{q}}^{0} \text { system }
\end{array}+\right.\text { h.c. }  \tag{5.7}\\
& H_{\mathrm{eff}}^{W W^{\prime}}=\frac{G_{F}^{2} M_{W}^{2}}{16 \pi^{2}} 2 \beta \Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}} \times \begin{cases}f_{\mathrm{LR}}(K) \bar{d}\left(1-\gamma_{5}\right) s \otimes \bar{d}\left(1+\gamma_{5}\right) s & K^{0}-\bar{K}^{0} \text { system } \\
f_{\mathrm{LR}}\left(B_{q}\right) \bar{q}\left(1-\gamma_{5}\right) b \otimes \bar{q}\left(1+\gamma_{5}\right) b & B_{q}^{0}-B_{\bar{q}}^{0} \text { system }\end{cases} \tag{5.8}
\end{align*}
$$

where $q=d, s$ and

$$
\begin{align*}
f_{\mathrm{LL}}(K)= & \left(\lambda_{c}^{\mathrm{LL}}(K)\right)^{2} \eta_{c c} \tilde{S}_{0}\left(x_{c}\right)+\left(\lambda_{t}^{\mathrm{LL}}(K)\right)^{2} \eta_{t t} \tilde{S}_{0}\left(x_{t}\right)+2 \lambda_{c}^{\mathrm{LL}}(K) \lambda_{t}^{\mathrm{LL}}(K) \eta_{c t} \tilde{S}_{0}\left(x_{c}, x_{t} \gamma 5.9\right) \\
f_{\mathrm{LL}}\left(B_{q}\right)= & \left(\lambda_{t}^{\mathrm{LL}}\left(B_{q}\right)\right)^{2} \eta_{B_{q}} \tilde{S}_{0}\left(x_{t}\right) \\
\tilde{S}_{0}(x)= & \left.\frac{x}{(1-x)^{2}}\left[\Delta_{1,1}^{4}+\frac{4 \Delta_{1,1}^{4}-16 \Delta_{L, 1}^{2}+1}{4} x+\frac{x^{2}}{4}+\frac{2 x \ln x}{1-x}\left(\Delta_{1,1}^{4}-\Delta_{1,1}^{2}-\frac{4 \Delta_{1,1}^{2}-1}{4} x\right)\right] 5.11\right) \\
\tilde{S}_{0}\left(x_{c}, x_{t}\right)= & x_{c} x_{t}\left[\frac{1}{\left(1-x_{c}\right)\left(1-x_{t}\right)}\left(\Delta_{1,1}^{4}-2 \Delta_{1,1}^{2}+\frac{1}{4}\right)\right. \\
& \quad+\frac{\ln x_{t}}{\left(x_{t}-x_{c}\right)\left(1-x_{t}\right)^{2}}\left(\Delta_{1,1}^{4}-2 \Delta_{1,1}^{2} x_{t}+\frac{x_{t}^{2}}{4}\right) \\
& \left.\quad+\frac{\ln x_{c}}{\left(x_{c}-x_{t}\right)\left(1-x_{c}\right)^{2}}\left(\Delta_{1,1}^{4}-2 \Delta_{1,1}^{2} x_{c}+\frac{x_{c}^{2}}{4}\right)\right] .  \tag{5.12}\\
f_{\mathrm{LR}}(K)= & \lambda_{c}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K) S_{c c}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K) S_{t t}(K) \\
& +\left(\lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right) S_{c t}(K)  \tag{5.13}\\
f_{\mathrm{LR}}\left(B_{q}\right)= & \left.f_{\mathrm{LR}}(K)\right|_{K \rightarrow B_{q}}  \tag{5.14}\\
S_{c c}(K)= & \frac{x_{c}}{\left(1-x_{c}\right)^{2}}\left[\left(4 \Delta_{1,1}^{2} \eta_{1}^{\mathrm{LR}}(K)-x_{c} \eta_{2}^{\mathrm{LR}}(K)\right)\left(1-x_{c}\right)\right.  \tag{5.15}\\
S_{t t}(K)= & \left.\left.S_{c c}(K)\right|_{c \rightarrow t} \quad+\left(4 \Delta_{1,1}^{2} \eta_{1}^{\mathrm{LR}}(K)-2 x_{c} \eta_{2}^{\mathrm{LR}}(K)+x_{c}^{2} \eta_{2}^{\mathrm{LR}}(K)\right) \ln x_{c}+\eta_{2}^{\mathrm{LR}}(K)\left(1-x_{c}\right)^{2} \ln \beta\right] \\
S_{c t}(K)= & \frac{\sqrt{x_{c} x_{t}}}{\left(1-x_{c}\right)\left(1-x_{t}\right)\left(x_{t}-x_{c}\right)}\left[x_{t}\left(4 \Delta_{1,1}^{2} \eta_{1}^{\mathrm{LR}}(K)-x_{t} \eta_{2}^{\mathrm{LR}}(K)\right)\left(1-x_{c}\right) \ln x_{t}\right. \tag{5.16}
\end{align*}
$$

with $x_{c}=m_{c}^{2} / M_{W}^{2}, x_{t}=m_{t}^{2} / M_{W}^{2}, \beta=M_{W}^{2} / M_{W^{\prime}}^{2}$. The next-to-leading-order QCD shortdistance corrections are $\eta_{c c}=1.38 \pm 0.20, \eta_{c t}=0.47 \pm 0.04, \eta_{t t}=0.57 \pm 0.01$ [38, 39], $\eta_{B_{d}}=0.551, \eta_{B_{s}}=0.837$ [40]. The QCD corrections are $\eta_{1}^{\mathrm{LR}}(K)=1.4, \eta_{2}^{\mathrm{LR}}(K)=1.17$ for $\Lambda_{\mathrm{QCD}}=0.2 \mathrm{GeV}$ 41] and $\eta_{1}\left(B_{q}\right) \simeq 1.8, \eta_{2}\left(B_{q}\right) \simeq 1.7$ at scale $m_{b}$ (42].

The matrix elements are given by

$$
\begin{align*}
\left\langle K^{0}\right| \bar{d} \gamma^{\mu}\left(1 \pm \gamma_{5}\right) s \otimes \bar{d} \gamma_{\mu}\left(1 \pm \gamma_{5}\right) s\left|\bar{K}^{0}\right\rangle & =\frac{4}{3} f_{K}^{2} m_{K} B_{K},  \tag{5.19}\\
\left\langle B_{q}^{0}\right| \bar{d} \gamma^{\mu}\left(1 \pm \gamma_{5}\right) s \otimes \bar{d} \gamma_{\mu}\left(1 \pm \gamma_{5}\right) s\left|\bar{B}_{q}^{0}\right\rangle & =\frac{4}{3} f_{B_{q}}^{2} m_{B_{q}} B_{B_{q}},  \tag{5.20}\\
\left\langle K^{0}\right| \bar{d}\left(1-\gamma_{5}\right) s \otimes \bar{d}\left(1+\gamma_{5}\right) s\left|\bar{K}^{0}\right\rangle & =\frac{1}{2 m_{K}}\left[\frac{1}{3}+\frac{2 m_{K}^{2}}{\left(m_{s}+m_{d}\right)^{2}}\right] f_{K}^{2} m_{K}^{2} B_{K}^{S},  \tag{5.21}\\
\left\langle B^{0}\right| \bar{q}\left(1-\gamma_{5}\right) b \otimes \bar{q}\left(1+\gamma_{5}\right) b\left|\bar{B}^{0}\right\rangle & =\frac{1}{2 m_{B_{q}}}\left[\frac{1}{3}+\frac{2 m_{B_{q}}^{2}}{m_{b}^{2}}\right] f_{B_{q}}^{2} m_{B_{q}}^{2} B_{B_{q}}^{S} \quad q=d, s . \tag{5.22}
\end{align*}
$$

The decay constant for neutral K meson is given by $f_{K} / f_{\pi}=1.198 \pm 0.003$ 43, 44 with $f_{\pi}=(130 \pm 5) \times 10^{-3} \mathrm{GeV}$ [35] and the bag parameter is $B_{K}=0.79 \pm 0.04 \pm 0.09$ [45]. For $B_{d}$ and $B_{s}$ mesons, $f_{B_{d}} \sqrt{B_{B_{d}}}=0.220 \pm 0.040 \mathrm{GeV}$ [40] and $f_{B_{s}} \sqrt{B_{B_{s}}}=0.221 \mathrm{GeV}$ [46] . The bag parameter from QCD sum rule gives $B_{B_{q}}^{S} / B_{B_{q}}=1.2 \pm 0.2$ [36].

For NU models, we must consider the generation dependence $\alpha$ in all $\Delta$ 's appeared in (4.14). To simplify the expressions, we denote the CKM factors as $V_{L}^{\alpha \beta} \equiv V_{L}^{\mathrm{CKM}, u_{\alpha} b_{\beta}}$, and

$$
\begin{equation*}
V_{L, 11}^{\alpha \beta}=\sum_{\alpha^{\prime}} V_{L}^{u, \alpha \alpha^{\prime}} \Delta_{1,1, \alpha^{\prime}} V_{L}^{d \dagger, \alpha^{\prime} \beta}, \quad V_{L, 12}^{\alpha \beta}=\sum_{\alpha^{\prime}} V_{L}^{u, \alpha \alpha^{\prime}} \Delta_{1,2, \alpha^{\prime}} V_{L}^{d \dagger, \alpha^{\prime} \beta} \tag{5.23}
\end{equation*}
$$

etc. Ignoring the higher order of $\Delta_{2,1, \alpha}$ and $\Delta_{2,2, \alpha}$, After tedious calculations, we get the effective Hamilton for $K^{0}-\bar{K}^{0}$ system in NU models as follows:

$$
\begin{align*}
H_{\mathrm{eff}}^{\mathrm{WW}}= & \frac{G_{F}^{2} M_{W}^{2}}{16 \pi^{2}}\left\{\left[\sum_{\alpha, \beta=u, c, t}\left(V_{L, 11}^{\alpha s} V_{L, 11}^{\alpha d *}\right)\left(V_{L, 11}^{\beta s} V_{L, 11}^{\beta d *}\right)+\frac{1}{4} \sum_{\alpha, \beta=u, c} x_{\alpha} x_{\beta}\left(V_{L, 11}^{\alpha s} V_{L}^{\alpha d *}\right)\left(V_{L}^{\beta s} V_{L, 11}^{\beta d *}\right)\right]\right. \\
& \left.\times I_{2}\left(x_{\alpha}, x_{\beta}, 1\right)-2 \sum_{\alpha, \beta=u, c} x_{\alpha} x_{\beta}\left(V_{L}^{\alpha s} V_{L}^{\alpha d *}\right)\left(V_{L}^{\beta s} V_{L}^{\beta d *}\right) I_{1}\left(x_{\alpha}, x_{\beta}, 1\right)\right\} \\
H_{\mathrm{eff}}^{W^{\prime} W^{\prime}}= & \frac{G_{F}^{2} M_{W}^{2}}{16 \pi^{2}}\left\{\gamma_{5}\right) s \otimes \bar{d} \gamma^{\mu}\left(1-\gamma_{5}\right) s+\text { h.c. }  \tag{5.24}\\
& \left(V_{L, \beta=u, c, t}^{\alpha s} V_{L, 12}^{\alpha d *}\right)\left(V_{L, 12}^{\beta s} V_{L, 12}^{\beta d *}\right) I_{2}\left(x_{\alpha}, x_{\beta}, 1\right) \\
& \left.+\frac{1}{4} \beta x_{t}^{2}\left(V_{L, 12}^{t s} V_{L}^{t d *}\right)\left(V_{L}^{t s} V_{L, 12}^{t d *}\right) I_{2}\left(x_{t}, 1\right)-2 \beta^{2} x_{t}^{2}\left(V_{L}^{t s} V_{L}^{t d *}\right)^{2} I_{1}\left(x_{t}, 1\right)\right\} \\
H_{\mathrm{eff}}^{W W^{\prime}=}= & \frac{G_{F}^{2} M_{W}^{2}}{8 \pi^{2}\left(g_{1}^{2} / g_{2}^{2}\right)} \beta\left\{\sum_{\alpha, \beta=u, c, t}\left[\left(V_{L, 11}^{\alpha s} V_{L, 12}^{\alpha d *}+V_{L, 12}^{\alpha s} V_{L, 11}^{\alpha d *}\right)\left(V_{L, 12}^{\beta s} V_{L, 11}^{\beta d *}+V_{L, 11}^{\beta s} V_{L, 12}^{\beta d *}\right)\right] I_{2}\left(x_{\alpha}, x_{\beta}, \beta\right)\right.  \tag{5.25}\\
& \left.-x_{t}^{2} \beta\left[\left(V_{L, 11}^{t s} V_{L}^{t * *}\right)\left(V_{L}^{t s} V_{L, 11}^{t d *}\right)\right] I_{1}\left(x_{t}, \beta\right)-\sum_{\alpha, \beta=u, c}\left(x_{\alpha} x_{\beta}\right)^{3 / 2}\left(V_{L}^{\alpha s} V_{L, 12}^{\alpha d *}\right)\left(V_{L, 12}^{\beta s} V_{L}^{\beta d *}\right)\right] \\
& \left.\times I_{1}\left(x_{\alpha}, x_{\beta}, \beta\right)\right\} \bar{d} \gamma_{\mu}\left(1-\gamma_{5}\right) s \otimes \bar{d} \gamma^{\mu}\left(1-\gamma_{5}\right) s+\text { h.c. } .
\end{align*}
$$

The effective Hamilton for $B^{0}-\bar{B}^{0}$ system can be obtained through the same procedure.

## 6. Constraints from neutral $K$ and $B$ system for $L R$ and LP models

In this section, we will concentrate on the constraints on our EWCL from mass differences in $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}, B_{s}^{0}-\bar{B}_{s}^{0}$ systems and indirect CP violation parameter $\left|\epsilon_{K}\right|$, mainly for LR and LP models. Due to complexity of CKM factors introduced in (5.23) for NU models, we will leave the investigation for NU model elsewhere.

The mass differences in $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}$ and $B_{s}^{0}-\bar{B}_{s}^{0}$ systems are determined by

$$
\begin{equation*}
\left.\Delta m_{K}=2 \operatorname{Re}\left\langle K^{0}\right| H_{\text {eff }}\left|\bar{K}^{0}\right\rangle \quad \Delta m_{B_{q}}=2\left|\left\langle B_{q}^{0}\right| H_{\text {eff }}\right| \bar{B}_{q}^{0}\right\rangle \mid \quad q=d, s \tag{6.1}
\end{equation*}
$$

and the indirect CP violation in K mesons can be expressed as

$$
\begin{equation*}
\left|\epsilon_{K}\right|=\frac{1}{2 \sqrt{2}}\left(\frac{\operatorname{Im}\left\langle K^{0}\right| H_{\text {eff }}\left|\bar{K}^{0}\right\rangle}{\operatorname{Re}\left\langle K^{0}\right| H_{\mathrm{eff}}\left|\bar{K}^{0}\right\rangle}+2 \xi_{0}\right) \approx \frac{\operatorname{Im}\left\langle K^{0}\right| H_{\mathrm{eff}}\left|\bar{K}^{0}\right\rangle}{\sqrt{2} \Delta m_{K}} \tag{6.2}
\end{equation*}
$$

where $\xi_{0}$ is the weak phase of $K \rightarrow \pi \pi$ decay amplitude with isospin zero. The pure $W$ contribution to mass differences in $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}$ and $B_{s}^{0}-\bar{B}_{s}^{0}$ systems and indirect CP violation in K mesons as functions of anomalous coupling $\Delta_{1,1}$ introduced in (A.1) is shown in figure 3 . From (4.11), we know that $\Delta_{1,1}$ characterize the anomalous coupling for charge current, it can deviate from 1 very much and therefore we choose region [0.8,1.2] for $\Delta_{1,1}$ as horizontal coordinate in figure 3 .

In numerical calculation, the input parameters are taken from particle data group (35) except those explicitly labeled.

$$
\begin{aligned}
G_{F} & =1.16637(1) \times 10^{-5} \mathrm{GeV}^{-2}, & M_{W} & =80.403 \pm 0.029 \mathrm{GeV}, \\
m_{K} & =(497.648 \pm 0.022) \times 10^{-3} \mathrm{GeV}, & \Delta m_{K}^{\exp } & =(3.483 \pm 0.006) \times 10^{-15} \mathrm{GeV}, \\
m_{d} & =5 \times 10^{-3} \mathrm{GeV}, & m_{s} & =95 \times 10^{-3} \mathrm{GeV}, \\
m_{c} & =1.25 \pm 0.09 \mathrm{GeV}, & m_{t} & =174.2 \pm 3.3 \mathrm{GeV}, \\
m_{B_{d}} & =(5279.4 \pm 0.5) \times 10^{-3} \mathrm{GeV}, & \Delta m_{B_{d}}^{\exp } & =(3.337 \pm 0.003) \times 10^{-13} \mathrm{GeV}, \\
m_{B_{s}} & =(5367.5 \pm 1.8) \times 10^{-3} \mathrm{GeV}, & \Delta m_{B_{s}}^{\exp } & =(1.17 \pm 0.003) \times 10^{-11} \mathrm{GeV},
\end{aligned}
$$

The CKM elements are given in terms of Wolfenstein parameterization (35):

$$
\lambda=0.2272, A=0.818, \bar{\rho}=0.221, \bar{\eta}=0.340,
$$

with the relations $s_{13} e^{i \delta}=\left(V_{L}^{u b}\right)^{*}=A \lambda^{3}(\rho+i \eta)=\frac{A \lambda^{3}(\bar{\rho}+i \bar{\eta}) \sqrt{1-A^{2} \lambda^{4}}}{\sqrt{1-\lambda^{2}}\left[1-A^{2} \lambda^{4}(\bar{\rho}+i \bar{\eta})\right]}$.
We find that for $\Delta m_{K},\left|\epsilon_{K}\right|, \Delta m_{B_{d}}$ and $\Delta m_{B_{s}}$, SM theoretical results ( $\Delta_{1,1}=1$ ) match to experiment values with error $33 \%, 18 \%, 6 \%$ and $23 \%$ respectively. These errors are expected from uncertainty of matrix elements and long distance contributions 47. New physics contributions must hide in these errors.

Up to the order liner in $\beta, W^{\prime}$ contribution to mass differences in $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}$, $B_{s}^{0}-\bar{B}_{s}^{0}$ systems and indirect CP violation in K mesons are

$$
\Delta m_{K}^{W W^{\prime}}=2 \operatorname{Re}\left\langle K^{0}\right| H_{\mathrm{eff}}^{W W^{\prime}}\left|\bar{K}^{0}\right\rangle
$$



Figure 3: Pure $W$ contribution to $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}, B_{s}^{0}-\bar{B}_{s}^{0}$ systems and indirect CP violation in K mesons. Anomalous coupling $\Delta_{1,1}=1$ corresponds to SM results which are explicitly written down in brackets.

$$
\begin{align*}
& =\frac{G_{F}^{2} M_{W}^{2} f_{K}^{2} m_{K} B_{K}^{S} \beta}{4 \pi^{2}} \Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}} \operatorname{Re}\left(f_{\mathrm{LR}}(K)\right)\left[\frac{1}{6}+\frac{m_{K}^{2}}{\left(m_{s}+m_{d}\right)^{2}}\right]  \tag{6.3}\\
\Delta m_{B_{q}}^{W W^{\prime}} & \left.=2\left|\left\langle B_{q}^{0}\right| H_{\mathrm{eff}}^{W W^{\prime}}\right| \bar{B}_{q}^{0}\right\rangle \mid \\
& =\frac{G_{F}^{2} M_{W}^{2} f_{B}^{2} m_{B_{q}} B_{B}^{S} \beta}{4 \pi^{2}} \Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}}\left|f_{\mathrm{LR}}\left(B_{q}\right)\right|\left[\frac{1}{6}+\frac{m_{B_{q}}^{2}}{m_{b}^{2}}\right]  \tag{6.4}\\
\left|\epsilon_{K}\right|^{W W^{\prime}} & \approx \frac{\operatorname{Im}\left\langle K^{0}\right| H_{\mathrm{eff}}^{W W^{\prime}}\left|\bar{K}^{0}\right\rangle}{\sqrt{2} \Delta m_{K}} \\
& =\frac{G_{F}^{2} M_{W}^{2} f_{K}^{2} m_{K} B_{K}^{S} \beta}{8 \sqrt{2} \pi^{2} \Delta m_{K}} \Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}} \operatorname{Im}\left(f_{\mathrm{LR}}(K)\right)\left[\frac{1}{6}+\frac{m_{K}^{2}}{\left(m_{s}+m_{d}\right)^{2}}\right] \tag{6.5}
\end{align*}
$$

For $W^{\prime}$ contributions, we discuss $K^{0}-\bar{K}^{0}, B^{0}-\bar{B}^{0}$ systems separately.

## $6.1 K^{0}-\bar{K}^{0}$ system

According to types of inner quark lines in the box diagrams, $W^{\prime}$ contributions to $\Delta m_{K}$


Figure 4: Ratio of $t t$ loop to experiment data for $W^{\prime}$ contribution to $K_{L}-K_{S}$ mass difference, $c c$ to $t t$ loop and $c t$ to $t t$ loop for $W^{\prime}$ contribution to $K_{L}-K_{S}$ mass difference in $K^{0}-\bar{K}^{0}$ system with solid blue line for $M_{W^{\prime}}=10 M_{W}$, dash red line for $M_{W^{\prime}}=15 M_{W}$, dash-dot pink line for $M_{W^{\prime}}=20 M_{W}$ and dot black line for $M_{W^{\prime}}=25 M_{W}$, respectively.
in (6.3) can be decomposed into $t t, c c$ and $c t$ quark loop contributions,

$$
\begin{align*}
\Delta m_{K}^{W W^{\prime}}=\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}} \Delta m_{K_{t t}}^{W W^{\prime}}[ & \operatorname{Re}\left[\lambda_{t}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)\right]+\operatorname{Re}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\Delta m_{K_{c c}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}} \\
& \left.+\operatorname{Re}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\Delta m_{K_{c t}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}}\right] \tag{6.6}
\end{align*}
$$

in which the CKM matrices are

$$
\begin{array}{ll}
\lambda_{x}^{\mathrm{LR}}(K) \lambda_{x}^{\mathrm{RL}}(K)=\left|V_{L}^{x s} V_{L}^{x d *} \bar{V}_{R}^{x s} \bar{V}_{R}^{x d *}\right| e^{-i\left(\alpha_{1}-\alpha_{2}-\beta_{1}-\phi_{x s}-\bar{\phi}_{x s}+\phi_{x d}+\bar{\phi}_{x d}\right)} & x=c, t  \tag{6.7}\\
\lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)=\left|V_{L}^{c s} \bar{V}_{R}^{c d *} \bar{V}_{R}^{t s} V_{L}^{t d *}\right| e^{-i\left(\alpha_{1}-\alpha_{3}-\beta_{1}+\beta_{2}-\phi_{c s}+\bar{\phi}_{c d}-\bar{\phi}_{t s}+\phi_{t d}\right)} & \arg \left(V_{L}^{\alpha \beta}\right)=\phi_{\alpha \beta} \\
\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)=\left|V_{L}^{t s} \bar{V}_{R}^{t d *} \bar{V}_{R}^{c s} V_{L}^{c d *}\right| e^{-i\left(\alpha_{1}-2 \alpha_{2}+\alpha_{3}-\beta_{1}-\beta_{2}-\phi_{t s}+\bar{\phi}_{t d}-\bar{\phi}_{c s}+\phi_{c d}\right)} & \arg \left(\bar{V}_{R}^{\alpha \beta}\right)=\bar{\phi}_{\alpha \beta}
\end{array}
$$

In figure 4 , we plot $\frac{\Delta m_{K_{t t}}^{W W^{\prime}}}{\Delta m_{K}^{\text {exp }}}, \frac{\Delta m_{K_{c c}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}}$ and $\frac{\Delta m_{K_{c t}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}}$ separately,
From figure 1 , we find that $\frac{\Delta m_{K_{t t}}^{W W^{\prime}}}{\Delta m_{K}^{\text {exp }}}$ is of order $10^{5}, \frac{\Delta m_{K K_{c c}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}}$ is of order $10^{-3}$ and $\frac{\Delta m_{K_{c t}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}}$ is of order $10^{-2}$. Therefore to reduce total contributions of $\Delta m_{K}^{W W^{\prime}}$, we have following four different kind of mechanisms

- Large $M_{W^{\prime}}$ : Take very large $W^{\prime}$ mass. This is traditional naive constraints to $W^{\prime}$ mass.
- Small $g_{2}$ : Take very small $W^{\prime}$ gauge coupling $g_{2} \ll g_{1}$. This can only happens if $f_{2} \gg g_{1} f_{1} / g_{2}$ to make large enough $W^{\prime}$ mass. Since two gauge couplings are not equal to each other, this situation is parity explicitly broken.


Figure 5: Ratio of $t t$ loop to experiment data for $W^{\prime}$ contribution to $\left|\epsilon_{K}\right|, c c$ to $t t$ loop and $c t$ to $t t$ loop for $W^{\prime}$ contribution to indirect CP violation in K mesons $\left|\epsilon_{K}\right|$ in $K^{0}-\bar{K}^{0}$ system with solid blue line for $M_{W^{\prime}}=10 M_{W}$, dash red line for $M_{W^{\prime}}=15 M_{W}$, dash-dot pink line for $M_{W^{\prime}}=20 M_{W}$ and dot black line for $M_{W^{\prime}}=25 M_{W}$, respectively.

- Small $\Delta_{2,1}$ : Take very small $\Delta_{2,1}$. This is the situation pointed out in our previous work [28]. Although realization of this situation in detail model is still lacking.
- Specific $V_{R}^{\mathrm{CKM}}$ : Choose special right hand CKM matrix elements to make $f_{\mathrm{LR}}(K)$ in (5.13) small. Numerically

$$
\begin{align*}
& \operatorname{Re}\left[\lambda_{t}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)\right]+\operatorname{Re}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\Delta m_{K_{c c}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}} \\
& \quad+\operatorname{Re}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\Delta m_{K_{c t}}^{W W^{\prime}}}{\Delta m_{K_{t t}}^{W W^{\prime}}} \ll 10^{-5} \tag{6.8}
\end{align*}
$$

Similar to $K_{L}-K_{S}$ mass difference, we can also decompose indirect CP violation parameter $\left|\epsilon_{K}\right|$ in K system as

$$
\begin{align*}
& \left.\left|\epsilon_{K}\right|^{W W^{\prime}}=\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}}\left|\epsilon_{K}\right|_{t t}^{W W^{\prime}} \right\rvert\, \operatorname{Im}\left[\lambda_{t}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)\right]+\operatorname{Im}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\left|\epsilon_{K}\right|_{c c}^{W W^{\prime}}}{\left.\left|\epsilon_{K}\right|\right|_{t t} ^{W W^{\prime}}} \\
& \left.+\operatorname{Im}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\left|\epsilon_{K}\right| c t}{\mid{ }_{c t} W^{\prime}} \right\rvert\,  \tag{6.9}\\
&\left|\epsilon_{K}\right|{ }_{t t}^{W W^{\prime}} \mid
\end{align*}
$$

In figure 5 , we plot $\frac{\left|\epsilon_{K}\right|{ }_{t} W^{\prime}}{\left|\epsilon_{K}\right|^{\mid e \times P}}, \frac{\left.\left|\epsilon_{K}\right|\right|_{c} W^{\prime}}{\left.\left|\epsilon_{K}\right|\right|_{t} ^{W} W^{\prime}}$ and $\frac{\left.\left|\epsilon_{K}\right|\right|_{t} W^{\prime}}{\left|\epsilon_{K}\right|{ }_{t t} W^{\prime}}$ separately,
From figure ${ }^{\text {a }}$, we find that $\frac{\left|\epsilon_{K}\right|{ }_{t}^{W W^{\prime}}}{\left|\epsilon_{K}\right|{ }^{\text {exP }}}$ is of order $10^{7}, \frac{\left.\left|\epsilon_{K}\right|\right|_{c} ^{W W^{\prime}}}{\left.\left|\epsilon_{K}\right|\right|_{t} ^{W} W^{\prime}}$ is of order $10^{-3}$ and $\frac{\left.\left|\epsilon_{K}\right|\right|_{t} W^{\prime}}{\left.\left|\epsilon_{K}\right|\right|_{t t} ^{W W^{\prime}}}$ is of order $10^{-2}$. To reduce total contributions of $\left|\epsilon_{K}\right|^{W W^{\prime}}$, we also can take either large
$M_{W^{\prime}}$; or small $g_{2}$; or small $\Delta_{2,1}$; or specific $V_{R}^{\text {CKM }}$ satisfying

$$
\begin{align*}
& \operatorname{Im}\left[\lambda_{t}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\operatorname{Im}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\left|\epsilon_{K}\right|_{c c}^{W W^{\prime}}}{\left|\epsilon_{K}\right|_{t t}^{W W^{\prime}}}\right. \\
& \quad+\operatorname{Im}\left[\lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)\right] \frac{\left|\epsilon_{K}\right|_{c t}^{W W^{\prime}}}{\left|\epsilon_{K}\right|_{t t}^{W W^{\prime}}} \ll 10^{-7} \tag{6.10}
\end{align*}
$$

The relation (6.8) and (6.10) offer constraints for right hand CKM matrix elements, as long as they really take the role of suppressing contribution from $W^{\prime}$ boson. If constraints (6.8) and (6.10) can not be satisfied, we must adjust $M_{W^{\prime}}, g_{2}$ and $\Delta_{2,1}$ to suppress contribution of $W^{\prime}$. To quantitatively estimate constraints for $M_{W^{\prime}}, g_{2}$ and $\Delta_{2,1}$, we take a special pseudomanifest left-right symmetric situation as an example. In this situation, $\bar{V}_{R}^{\alpha \beta}=\left(V_{L}^{\alpha \beta}\right)^{*}$, which implies the relations $\phi_{\alpha \beta}=-\bar{\phi}_{\alpha \beta}$ between phases defined in (6.7). Then CKM factors appeared in (6.6) and (6.9) can be simplified as

$$
\begin{gathered}
\lambda_{c}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)=\left|V_{L}^{c s} V_{L}^{c d}\right|^{2} e^{-i\left(\alpha_{1}-\alpha_{2}-\beta_{1}\right)} \quad \lambda_{t}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)=\left|V_{L}^{t s} V_{L}^{t d}\right|^{2} e^{-i\left(\alpha_{1}-\alpha_{2}-\beta\right.}(\bar{b} .1] \\
\lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K)=2\left|V_{L}^{c s} V_{L}^{c d} V_{L}^{t s} V_{L}^{t d}\right|\left[\cos \left(\alpha_{1}-\alpha_{2}-\beta_{1}\right) \cos \left(\alpha_{2}-\alpha_{3}+\beta_{2}\right)\right. \\
\left.-i \sin \left(\alpha_{1}-\alpha_{2}-\beta_{1}\right) \cos \left(\alpha_{2}-\alpha_{3}+\beta_{2}\right)\right]
\end{gathered}
$$

Notice that constraint from $\left|\epsilon_{K}\right|$ demands imaginary part of above CKM matrix elements must at least two order of magnitude smaller than their real part, this leads us to take following choice of phase angle

$$
\begin{equation*}
\alpha_{1}-\alpha_{2}-\beta_{1}=0 . \tag{6.12}
\end{equation*}
$$

Then the imaginary part of all CKM matrix elements in (6.11) will vanish and the cc, tt and ct loops do not contribute to $\left|\epsilon_{K}\right|{ }^{W W^{\prime}}$ separately. This special choice of phase angle is originally proposed in ref. [37] which directly leads to

$$
\begin{equation*}
\left|\epsilon_{K}\right|^{W W^{\prime}}=0 . \tag{6.13}
\end{equation*}
$$

The values of CKM matrix factors in (6.11) now can be worked out in terms of left hand CKM matrix in [35],

$$
\begin{align*}
& \lambda_{c}^{\mathrm{LR}}(K) \lambda_{t}^{\mathrm{RL}}(K)+\lambda_{t}^{\mathrm{LR}}(K) \lambda_{c}^{\mathrm{RL}}(K) \mid \underset{\text { manifest }}{\stackrel{\alpha_{1}-\alpha_{2}}{=}=}==\beta_{1}=0.00107 \cos \left(\alpha_{2}-\alpha_{3}+\beta_{2}\right) \tag{6.14}
\end{align*}
$$

Now, except an overall factor $\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}}, \frac{\Delta m_{K}^{W} W^{\prime}}{\Delta m_{K}^{\text {exp }}}$. depends on two other parameters, $\Delta_{1,1}$ and $\cos \left(\alpha_{2}-\alpha_{3}+\beta_{2}\right)$. Considering that anomalous coupling $\Delta_{1,1}$ can not deviate from 1 very much, in figure 6, we plot $\frac{\Delta m_{K}^{W W^{\prime}}}{\Delta m_{K}^{\mathrm{ex}}} /\left(\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}}\right)$ as function of $\cos \left(\alpha_{2}-\alpha_{3}+\beta_{2}\right)$ with anomalous coupling $\Delta_{1,1}=1$.

From figure 6, we find if $\Delta_{2,1} \sim 1$ and $g_{2} \sim g_{1}$, then to make $\frac{\Delta m_{K}^{W W^{\prime}}}{\Delta m_{K}^{\mathrm{exp}}} \ll 1$, we must have $M_{W^{\prime}} \sim$ several TeVs. This is the naive prediction of $W^{\prime}$ mass in traditional left-right


Figure 6: $\frac{\Delta m_{K}^{W W^{\prime}}}{\Delta m_{K}^{\text {exp }}} /\left(\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}}\right)$ as function of $\cos \left(\alpha_{2}-\alpha_{3}+\beta_{2}\right)$ with anomalous coupling $\Delta_{1,1}=1$.
symmetric models. While if $M_{W^{\prime}}$ is at order of 1 TeV , to make $\frac{\Delta m_{K}^{W W^{\prime}}}{\Delta m_{K}^{\text {exp }}} \ll 1$, we must have $\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}} \ll 10^{-1}$ which demands either very small anomalous coupling $\Delta_{2,1}$ or small gauge coupling $g_{2}$. Note that from (4.13), unlike $\Delta_{1,1}$ which roughly is 1 since it is anomalous coupling of charged current for $W$ boson, $\Delta_{2,1}$ is anomalous coupling of charged current for $W^{\prime}$ boson and there is no experiment constraint on its value. This provides us an alternative way to reduce $W^{\prime}$ contribution. This possibility was first pointed out in our previous work [28] where $\Delta_{2,1}$ is denoted by $\Delta_{R, 1}$.

## 6.2 $B^{0}-\bar{B}^{0}$ system

For $B_{d}^{0}-\bar{B}_{d}^{0}$ system, similar to $K^{0}-\bar{K}^{0}$ system, we can decompose corresponding effective Hamiltonian as,

$$
\begin{array}{r}
\Delta m_{B_{d}}^{W W^{\prime}}=\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}} \Delta m_{B_{d} t t}^{W W^{\prime}} \left\lvert\, \lambda_{t}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{d}\right)+\lambda_{c}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{d}\right) \frac{\Delta m_{B_{d} c c}^{W W^{\prime}}}{\Delta m_{B_{d} t t}^{W W^{\prime}}}\right. \\
\left.+\left[\lambda_{c}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{d}\right)+\lambda_{t}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{d}\right)\right] \frac{\Delta m_{B_{d} c t}^{W W^{\prime}}}{\Delta m_{B_{d} t t}^{W W^{\prime}}} \right\rvert\,  \tag{6.15}\\
\lambda_{x}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{x}^{\mathrm{RL}}\left(B_{d}\right)=\left|V_{L}^{x b} V_{L}^{x d *} \bar{V}_{R}^{x b} \bar{V}_{R}^{x d *}\right| e^{-i\left(\alpha_{1}-\alpha_{3}-\beta_{1}-\beta_{2}-\phi_{x b}-\bar{\phi}_{x b}+\phi_{x d}+\bar{\phi}_{x d}\right) \quad x=c, t}
\end{array}
$$



Figure 7: Ratio of $t t$ loop to experiment data for $W^{\prime}$ contribution to $\Delta m_{B_{d}}, c c$ to $t t$ loop and $c t$ to $t t$ loop for $W^{\prime}$ contribution to mass difference in $B_{d}^{0}-\bar{B}_{d}^{0}$ system with solid blue line for $M_{W^{\prime}}=10 M_{W}$, dash red line for $M_{W^{\prime}}=15 M_{W}$, dash-dot pink line for $M_{W^{\prime}}=20 M_{W}$ and dot black line for $M_{W^{\prime}}=25 M_{W}$, respectively.

$$
\begin{array}{ll}
\lambda_{c}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{d}\right)=\left|V_{L}^{c b} \bar{V}_{R}^{c d *} \bar{V}_{R}^{t b} V_{L}^{t d *}\right| e^{-i\left(\alpha_{1}+\alpha_{2}-2 \alpha_{3}-\beta_{1}-\phi_{c b}+\bar{\phi}_{c d}-\bar{\phi}_{t b}+\phi_{t d}\right)} & \arg \left(V_{L}^{\alpha \beta}\right)=\phi_{\alpha \beta} \\
\lambda_{t}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{d}\right)=\left|V_{L}^{t b} \bar{V}_{R}^{t d *} \bar{V}_{R}^{c b} V_{L}^{c d *}\right| e^{-i\left(\alpha_{1}-\alpha_{2}-\beta_{1}-2 \beta_{2}-\phi_{t b}+\bar{\phi}_{t d}-\bar{\phi}_{c b}+\phi_{c d}\right)} & \arg \left(\bar{V}_{R}^{\alpha \beta}\right)=\bar{\phi}_{\alpha \beta}
\end{array}
$$

In figure 7. we plot $\frac{\Delta m_{B_{d} t^{t t}}^{W}}{\Delta m_{B_{d}}^{\text {exp }}}, \frac{\Delta m_{B_{d} c^{c c}}^{W} W^{\prime}}{\Delta m_{B_{d} t t}^{W W^{\prime}}}$ and $\frac{\Delta m_{B_{d} c t}^{W} W^{\prime}}{\Delta m_{B_{d} t t}^{W} W^{\prime}}$ separately,
From figure 7, we find that $\frac{\Delta m_{B_{d}}^{W} W^{\prime}}{\Delta m_{B_{d}}^{d x p}}$ is of order $10^{5}, \frac{\Delta m_{B_{d} c^{c}}^{W W^{\prime}}}{\Delta m_{B_{d} t t}^{W} W^{\prime}}$ is of order $10^{-3}$ and $\frac{\Delta m_{B_{d} c^{\prime}}^{W} W^{\prime}}{\Delta m_{B_{d} t t}^{W} W^{\prime}}$ is of order $10^{-2}$. To reduce total contributions of $\frac{\Delta m_{B_{d}}^{W W^{\prime}}}{\Delta m_{B_{d}}^{\text {exp }}}$, we can take either large $M_{W^{\prime}}$; or small $g_{2}$; or small $\Delta_{2,1}$; or specific $V_{R}^{\mathrm{CKM}}$ which satisfy

$$
\begin{align*}
& \left\lvert\, \lambda_{t}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{d}\right)+\lambda_{c}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{d}\right) \frac{\Delta m_{B_{d} c c}^{W W^{\prime}}}{\Delta m_{B_{d} t t}^{W W^{\prime}}}\right. \\
& \left.\quad+\left[\lambda_{c}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{d}\right)+\lambda_{t}^{\mathrm{LR}}\left(B_{d}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{d}\right)\right] \frac{\Delta m_{B_{d} c t}^{W W^{\prime}}}{\Delta m_{B_{d} t t}^{W W^{\prime}}} \right\rvert\, \ll 10^{-5} \tag{6.16}
\end{align*}
$$

For $B_{s}^{0}-\bar{B}_{s}^{0}$ system,

$$
\begin{array}{r}
\Delta m_{B_{s}}^{W W^{\prime}}=\Delta_{2,1}^{2} \frac{g_{2}^{2}}{g_{1}^{2}} \Delta m_{B_{s} t t}^{W W^{\prime}} \left\lvert\, \lambda_{t}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{s}\right)+\lambda_{c}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{s}\right) \frac{\Delta m_{B_{s} c c}^{W W^{\prime}}}{\Delta m_{B_{s} t t}^{W W^{\prime}}}\right. \\
\left.+\left[\lambda_{c}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{s}\right)+\lambda_{t}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{s}\right)\right] \frac{\Delta m_{B_{s} c t}^{W}}{\Delta m_{B_{s} t t}^{W} W^{\prime}} \right\rvert\,(6  \tag{6.17}\\
\lambda_{x}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{x}^{\mathrm{RL}}\left(B_{s}\right)=\left|V_{L}^{x b} V_{L}^{x s *} \bar{V}_{R}^{x b} \bar{V}_{R}^{x s *}\right| e^{-i\left(\alpha_{2}-\alpha_{3}-\beta_{2}-\phi_{x b}-\bar{\phi}_{x b}+\phi_{x s}+\bar{\phi}_{x s}\right)} \quad x=c, t
\end{array}
$$



Figure 8: Ratio of $t t$ loop to experiment data for $W^{\prime}$ contribution to $\Delta m_{B_{s}}, c c$ to $t t$ loop and $c t$ to $t t$ loop for $W^{\prime}$ contribution to mass difference in $B_{s}^{0}-\bar{B}_{s}^{0}$ system with solid blue line for $M_{W^{\prime}}=10 M_{W}$, dash red line for $M_{W^{\prime}}=15 M_{W}$, dash-dot pink line for $M_{W^{\prime}}=20 M_{W}$ and dot black line for $M_{W^{\prime}}=25 M_{W}$, respectively.

$$
\begin{array}{ll}
\lambda_{c}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{s}\right)=\left|V_{L}^{c b} \bar{V}_{R}^{c s *} \bar{V}_{R}^{t b} V_{L}^{t s *}\right| e^{-i\left(2 \alpha_{2}-2 \alpha_{3}-\phi_{c b}+\bar{\phi}_{c s}-\bar{\phi}_{t b}+\phi_{t s}\right)} & \arg \left(V_{L}^{\alpha \beta}\right)=\phi_{\alpha \beta} \\
\lambda_{t}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{s}\right)=\left|V_{L}^{t b} \bar{V}_{R}^{t s *} \bar{V}_{R}^{c b} V_{L}^{c s *}\right| e^{-i\left(-2 \beta_{2}-\phi_{t b}+\bar{\phi}_{t s}-\bar{\phi}_{c b}+\phi_{c s}\right)} & \arg \left(\bar{V}_{R}^{\alpha \beta}\right)=\bar{\phi}_{\alpha \beta}
\end{array}
$$

In figure 8, we plot $\frac{\Delta m_{B_{s t t}}^{W W^{\prime}}}{\Delta m_{B_{s}}^{\mathrm{sPP}}}, \frac{\Delta m_{B_{s c c}}^{W W^{\prime}}}{\Delta m_{B_{s} t t}^{W W^{\prime}}}$ and $\frac{\Delta m_{B_{s} c t}^{W W^{\prime}}}{\Delta m_{B_{s} t t}^{W W^{\prime}}}$ separately,
From figure 8, we find that $\frac{\Delta m_{B_{s}+t}^{W} W^{\prime}}{\Delta m_{B_{s}}^{\text {exp }}}$ is of order $10^{4}, \frac{\Delta m_{B_{s c c}}^{W W^{\prime}}}{\Delta m_{B_{s} s t}^{W} W^{\prime}}$ is of order $10^{-3}$ and $\frac{\Delta m_{B_{s c t}}^{W W^{\prime}}}{\Delta m_{B_{s} t t}^{W W^{\prime}}}$ is of order $10^{-2}$. To reduce total contributions of $\frac{\Delta m_{B_{s}}^{W W^{\prime}}}{\Delta m_{B s}^{\text {exp }}}$, we can also take either large $M_{W^{\prime}}$; or small $g_{2}$; or small $\Delta_{2,1}$; or specific $V_{R}^{\mathrm{CKM}}$ which satisfy

$$
\begin{align*}
& \left\lvert\, \lambda_{t}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{s}\right)+\lambda_{c}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{s}\right) \frac{\Delta m_{B_{s} c c}^{W W^{\prime}}}{\Delta m_{B_{s} t t}^{W W^{\prime}}}\right. \\
& \left.\quad+\left[\lambda_{c}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{t}^{\mathrm{RL}}\left(B_{s}\right)+\lambda_{t}^{\mathrm{LR}}\left(B_{s}\right) \lambda_{c}^{\mathrm{RL}}\left(B_{s}\right)\right] \frac{\Delta m_{B_{s} c t}^{W W^{\prime}}}{\Delta m_{B_{s} t t}^{W W^{\prime}}} \right\rvert\, \ll 10^{-4} \tag{6.18}
\end{align*}
$$

Now we come to Higgs contribution to $H_{\text {eff }}$. This part is universal and from vertices given by (4.7), the amplitude of the diagram mediated by neutral Higgs $\tilde{h}$ is

$$
M^{\tilde{h}}=\frac{1}{4 m_{h}^{2}}\left\{\begin{array}{l}
\bar{d}\left(A_{d}^{d s}+B_{d}^{d s} \gamma^{5}\right) s \otimes \bar{d}\left(A_{d}^{d s}+B_{d}^{d s} \gamma^{5}\right) s K^{0}-\bar{K}^{0} \text { system }  \tag{6.19}\\
\bar{q}\left(A_{d}^{q b}+B_{d}^{q b} \gamma^{5}\right) b \otimes \bar{q}\left(A_{d}^{q b}+B_{d}^{q b} \gamma^{5}\right) b \quad B_{q}^{0}-\bar{B}_{q}^{0} \text { system } \quad q=d, s
\end{array}\right.
$$

and Higgs contribution to $H_{\text {eff }}$ is related to the amplitude by $H_{\text {eff }}^{\tilde{h}}=\frac{1}{2} M^{\tilde{h}}$, it's matrix element is

$$
\begin{equation*}
\left\langle K^{0}\right| H_{\mathrm{eff}}^{\tilde{h}}\left|\bar{K}^{0}\right\rangle=\frac{1}{8 m_{h}^{2}}\left\langle K^{0}\right| \bar{d}\left(A_{d}^{d s}+B_{d}^{d s} \gamma^{5}\right) s \otimes \bar{d}\left(A_{d}^{d s}+B_{d}^{d s} \gamma^{5}\right) s\left|\bar{K}^{0}\right\rangle \tag{6.20}
\end{equation*}
$$

$$
\begin{align*}
& =\left[\left(\tilde{y}_{d}^{d s}-\tilde{y}_{d}^{\dagger d s}\right)^{2}-\left(\tilde{y}_{d}^{d s}+\tilde{y}_{d}^{\dagger d s}\right)^{2}+\frac{\left(\left(\tilde{y}_{d}^{d s}+\tilde{y}_{d}^{\dagger d s}\right)^{2}-11\left(\tilde{y}_{d}^{d s}-\tilde{y}_{d}^{\dagger d s}\right)^{2}\right) m_{K}^{2}}{\left(m_{s}+m_{d}\right)^{2}}\right] \frac{f_{K}^{2} m_{K} B_{K}^{S}}{96 m_{h}^{2}} \\
\left\langle B_{q}^{0}\right| H_{\mathrm{eff}}^{\tilde{h}}\left|B_{\bar{q}}^{0}\right\rangle & =\frac{1}{8 m_{h}^{2}}\left\langle B_{q}^{0}\right| \bar{q}\left(A_{d}^{q b}+B_{d}^{q b} \gamma^{5}\right) b \otimes \bar{q}\left(A_{d}^{q b}+B_{d}^{q b} \gamma^{5}\right) b\left|B_{\bar{q}}^{0}\right\rangle \quad q=d, s  \tag{6.21}\\
& =\left[\left(\tilde{y}_{d}^{q b}-\tilde{y}_{d}^{\dagger q b}\right)^{2}-\left(\tilde{y}_{d}^{q b}+\tilde{y}_{d}^{\dagger q b}\right)^{2}+\frac{\left(\left(\tilde{y}_{d}^{q b}+\tilde{y}_{d}^{\dagger q b}\right)^{2}-11\left(\tilde{y}_{d}^{q b}-\tilde{y}_{d}^{\dagger q b}\right)^{2}\right) m_{B_{q}}^{2}}{\left(m_{s}+m_{d}\right)^{2}}\right] \frac{f_{B_{q}}^{2} m_{B_{q}} B_{B_{q}}^{S}}{96 m_{h}^{2}}
\end{align*}
$$

In the case of SM, above matrix elements vanishes due to non-existence of flavor changing coulings. Then in SM, there is no Higgs contribution to $\Delta m_{K}^{h}, \Delta m_{B_{q}}^{h}$ and $\left|\epsilon_{K}\right|^{h}$. Beyond SM Higgs, demanding Higgs contributions to $\Delta m_{K} \Delta m_{B}$ are much smaller than their experimental value, we find constraints

$$
\begin{align*}
\operatorname{Re}\left[\left(\tilde{y}_{d}^{d s}+\tilde{y}_{d}^{\dagger d s}\right)^{2}-11.4\left(\tilde{y}_{d}^{d s}-\tilde{y}_{d}^{\dagger d s}\right)^{2}\right] & \ll 6.45 \times 10^{-7}\left(\frac{m_{h}}{1 \mathrm{TeV}}\right)^{2}  \tag{6.22}\\
\left|\left(\tilde{y}_{d}^{d b}+\tilde{y}_{d}^{\dagger d b}\right)^{2}-11.4\left(\tilde{y}_{d}^{d b}-\tilde{y}_{d}^{\dagger d b}\right)^{2}\right| & \ll 1.74 \times 10^{-6}\left(\frac{m_{h}}{1 \mathrm{TeV}}\right)^{2}  \tag{6.23}\\
\left|\left(\tilde{y}_{d}^{s b}+\tilde{y}_{d}^{\dagger s b}\right)^{2}-11.4\left(\tilde{y}_{d}^{s b}-\tilde{y}_{d}^{\dagger s b}\right)^{2}\right| & \ll 6.10 \times 10^{-5}\left(\frac{m_{h}}{1 \mathrm{TeV}}\right)^{2} \tag{6.24}
\end{align*}
$$

While demanding Higgs contributions to $\left|\epsilon_{K}\right|$ is much smaller than its experimental value, we find constraint:

$$
\begin{equation*}
\operatorname{Im}\left[\left(\tilde{y}_{d}^{d s}+\tilde{y}_{d}^{\dagger d s}\right)^{2}-11.4\left(\tilde{y}_{d}^{d s}-\tilde{y}_{d}^{\dagger d s}\right)^{2}\right] \ll 0.0065 \times \operatorname{Re}\left[\left(\tilde{y}_{d}^{d s}+\tilde{y}_{d}^{\dagger d s}\right)^{2}-11.4\left(\tilde{y}_{d}^{d s}-\tilde{y}_{d}^{\dagger d s}\right)^{2}\right]( \tag{6.25}
\end{equation*}
$$

which imply term $\left(\tilde{y}_{d}^{d s}+\tilde{y}_{d}^{\dagger d s}\right)^{2}-11.4\left(\tilde{y}_{d}^{d s}-\tilde{y}_{d}^{\dagger d s}\right)^{2}$ is approximately real. Further combing constraint (6.22),

$$
\begin{equation*}
\left(\tilde{y}_{d}^{d s}+\tilde{y}_{d}^{\dagger d s}\right)^{2}-11.4\left(\tilde{y}_{d}^{d s}-\tilde{y}_{d}^{\dagger d s}\right)^{2} \ll 6.45 \times 10^{-7}\left(\frac{m_{h}}{1 \mathrm{TeV}}\right)^{2} \tag{6.26}
\end{equation*}
$$

## 7. Summary

In this paper, we have presented the complete list of electroweak chiral Lagrangian for $W^{\prime}, Z^{\prime}$, a neutral light higgs and those discovered SM particles with symmetry $\mathrm{SU}(2)_{1} \otimes \mathrm{SU}(2)_{2} \otimes \mathrm{U}(1)$. The bosonic part is accurate up to order of $p^{4}$. The matter part involving various fermions representation arrangements such as LR, LP, HP, FP, UN, NU includes dimension three Yukawa type and dimension four gauge type operators. The gauge boson and fermion mixings and masses are universal. For $W-W^{\prime}$ mixing, there exists two independent parameters just accounting two physical quantities, mixing angle $\zeta$ and mass ratio $M_{W} / M_{W^{\prime}}$. For neutrino mixing, existence of three heavy neutrinos will violate unitarity of rotation matrix among three light neutrinos. The left and right hand quark mixings lead to left and right hand CKM matrices with right hand CKM matrix parameterized in the same structure of left hand one multiplying five extra phase angles. We express Goldstone, Higgs and gauge couplings to quarks in gauge boson and fermion mass eigenstates. We build up effective Hamiltonian for neutral $K$ and $B$ systems and perform detail calculations for LR and LP models to mass differences for $K^{0}-\bar{K}^{0}, B_{d}^{0}-\bar{B}_{d}^{0}$ and $B_{s}^{0}-\bar{B}_{s}^{0}$ systems and indirect CP violation parameter $\epsilon_{K}$ for $K$ mesons. We show that
just $W$ itself with anomalous coupling $\Delta_{1,1}$ near to 1 is already account for experiment data. Except the case of mutual cancelations and very heavy $W^{\prime}$ up to order of $100 M_{W}$, there are other three ways to suppress $W^{\prime}$ contributions: small right hand gauge coupling $g_{2}$, small anomalous coupling $\Delta_{2,1}$, or special combination of CKM matrix elements. The smallness of higgs contribution leads to some constraints on $h \bar{q} q$ couplings.

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## A. Anomalous gauge couplings $\Delta$

For quark,

$$
\begin{align*}
& \Delta_{1,1, \alpha}= \begin{cases}1-\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha} & \text { LR,LP } \\
1-\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha} & \text { HP,FP } \\
0 & \text { UN } \\
\left(1-\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases}  \tag{A.1}\\
& \Delta_{1,2, \alpha}= \begin{cases}\delta_{R, 2, \alpha}-\delta_{R, 6, \alpha} & \text { LR,LP } \\
0 & \text { HP,FP } \\
1-\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha} & \text { UN } \\
0 \delta_{\alpha \alpha_{1}}+\left(1-\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha}\right) \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{1,1, \alpha}^{3}= \begin{cases}\left(1-\delta_{L, 1, \alpha}+\delta_{L, 4, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}+\delta_{L, 7, \alpha} & \text { LR,LP } \\
\left(1-\delta_{L, 1, \alpha}+\delta_{L, 4, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}+\delta_{L, 7, \alpha} & \text { HP,FP } \\
0 & \text { UN } \\
\left(\left(1-\delta_{L, 1, \alpha}+\delta_{L, 4, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}+\delta_{L, 7, \alpha}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{1,2, \alpha}^{3}= \begin{cases}\left(\delta_{R, 2, \alpha}+\delta_{R, 6, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{R, 5, \alpha} & \text { LR,LP } \\
0 & \text { HP,FP } \\
\left(1-\delta_{L, 1, \alpha}+\delta_{L, 4, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}+\delta_{L, 7, \alpha} & \text { UN } \\
0 \delta_{\alpha \alpha_{1}}+\left[\left(1-\delta_{L, 1, \alpha}+\delta_{L, 4, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}+\delta_{L, 7, \alpha}\right] \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{1, \alpha}= \begin{cases}\left(\delta_{L, 1, \alpha}-\delta_{R, 2, \alpha}-\delta_{L, 4, \alpha}-\delta_{R, 6, \alpha}+\frac{2}{3} \delta_{L, 7, \alpha}\right) \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{L, 3, \alpha}-\delta_{R, 5, \alpha} & \text { LR,LP } \\
\left(\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha}+\frac{2}{3} \delta_{L, 7, \alpha}\right) \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{L, 3, \alpha} & \text { HP,FP } \\
\left(\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha}+\frac{2}{3} \delta_{L, 7, \alpha}\right) \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{L, 3, \alpha} & \text { UN } \\
\left(\delta_{L, 1, \alpha}-\delta_{L, 4, \alpha}+\frac{2}{3} \delta_{L, 7, \alpha}\right) \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{L, 3, \alpha} & \text { NU }\end{cases}  \tag{A.2}\\
& \Delta_{2,1, \alpha}= \begin{cases}1-\delta_{R, 1, \alpha}-\delta_{R, 4, \alpha} & \text { LR,LP } \\
\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha} & \text { HP,FP } \\
0 & \text { UN } \\
\left(\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases}
\end{align*}
$$

$$
\begin{aligned}
& \Delta_{2,2, \alpha}= \begin{cases}\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha} & \text { LR,LP } \\
0 & \text { HP,FP } \\
\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha} & \text { UN } \\
0 \delta_{\alpha \alpha_{1}}+\left(\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}\right) \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{2,1, \alpha}^{3}= \begin{cases}\left(1-\delta_{R, 1, \alpha}+\delta_{R, 4, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{R, 3, \alpha}+\delta_{R, 7, \alpha} & \text { LR,LP } \\
\left(\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}+\delta_{L, 6, \alpha} & \text { HP,FP } \\
0 & \text { UN } \\
\left(\left(\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}+\delta_{L, 6, \alpha}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{2,2, \alpha}^{3}= \begin{cases}\left(\delta_{L, 2, \alpha}+\delta_{L, 6, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha} & \text { LR,LP } \\
0 & \text { HP,FP } \\
\left(\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}\right) \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}+\delta_{L, 6, \alpha} & \text { UN } \\
0 \delta_{\alpha \alpha_{1}}+\left(\left(\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha} \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}+\delta_{L, 6, \alpha}\right) \delta_{\alpha \alpha_{2}}\right. & \text { NU }\end{cases} \\
& \Delta_{2, \alpha}=\left\{\begin{array}{lll}
\left(\delta_{R, 1, \alpha}-\delta_{L, 2, \alpha}-\delta_{R, 4, \alpha}-\delta_{L, 6, \alpha}+\frac{2}{3} \delta_{R, 7, \alpha} \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{R, 3, \alpha}-\delta_{L, 5, \alpha}\right. & \text { LR,LP } \\
\left(1-\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}+\frac{2}{3} \delta_{R, 7, \alpha} \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{L, 5, \alpha}+\delta_{R, 7, \alpha}\right. & \text { HP,FP } \\
\left(1-\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}+\frac{2}{3} \delta_{R, 7, \alpha} \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{L, 5, \alpha}+\delta_{R, 7, \alpha}\right. & \text { UN } \\
\left(1-\delta_{L, 2, \alpha}-\delta_{L, 6, \alpha}+\frac{2}{3} \delta_{R, 7, \alpha} \frac{\tau_{3}}{2}+\frac{1}{6}-\delta_{L, 5, \alpha}+\delta_{R, 7, \alpha}\right. & \text { NU }
\end{array}\right.
\end{aligned}
$$

For lepton:

$$
\begin{align*}
& \Delta_{1,1, \alpha}^{l}= \begin{cases}1-\delta_{L, 1, \alpha}^{l}-\delta_{L, 4, \alpha}^{l} & \text { LR,HP } \\
1-\delta_{L, 1, \alpha}^{l}-\delta_{L, 4, \alpha}^{l} & \text { LP,FP,UN } \\
\left(1-\delta_{L, 1, \alpha}^{l}-\delta_{L, 4, \alpha}^{l}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{1,2, \alpha}^{l}= \begin{cases}\delta_{R, 2, \alpha}^{l}-\delta_{R, 6, \alpha}^{l} & \text { LR,HP } \\
0 & \text { LP,FP,UN } \\
0 \delta_{\alpha \alpha_{1}}+\left(1-\delta_{L, 1, \alpha}^{l}-\delta_{L, 4, \alpha}^{l}\right) \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{1,1, \alpha}^{l 3}= \begin{cases}\left(1-\delta_{L, 1, \alpha}^{l}+\delta_{L, 4, \alpha}^{l}\right) \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}^{l}+\delta_{L, 7, \alpha}^{l} & \text { LR,HP } \\
\left(1-\delta_{L, 1, \alpha}^{l}+\delta_{L, 4, \alpha}^{l} \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}^{l}+\delta_{L, 7, \alpha}^{l}\right. & \text { LP,FP,UN } \\
\left(\left(1-\delta_{L, 1, \alpha}^{l}+\delta_{L, 4, \alpha}^{l} \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}^{l}+\delta_{L, 7, \alpha}^{l}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}}\right. \text { NU }\end{cases} \\
& \Delta_{1,2, \alpha}^{l 3}= \begin{cases}\left(\delta_{R, 2, \alpha}^{l}+\delta_{R, 6, \alpha}^{l}\right) \frac{\tau^{3}}{2}+\delta_{R, 5, \alpha}^{l} & \text { LR,HP } \\
0 & \text { LP,FP,UN } \\
0 \delta_{\alpha \alpha_{1}}+\left(\left(1-\delta_{L, 1, \alpha}^{l}+\delta_{L, 4, \alpha}^{l}\right) \frac{\tau^{3}}{2}+\delta_{L, 3, \alpha}^{l}+\delta_{L, 7, \alpha}^{l}\right) \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{1, \alpha}^{l}= \begin{cases}\left(\delta_{L, 1, \alpha}^{l}-\delta_{R, 2, \alpha}^{l}-\delta_{L, 4, \alpha}^{l}-\delta_{R, 6, \alpha}^{l}-2 \delta_{L, 7, \alpha}^{l}\right) \frac{\tau_{3}}{2}-\frac{1}{2}-\delta_{L, 3, \alpha}^{l}-\delta_{R, 5, \alpha}^{l} & \text { LR,HP } \\
\left(\delta_{L, 1, \alpha}^{l}-\delta_{L, 4, \alpha}^{l}-2 \delta_{L, 7, \alpha}^{l} \frac{\tau_{3}}{2}-\frac{1}{2}-\delta_{L, 3, \alpha}^{l}\right. & \text { LP,FP,UN } \\
\left(\delta_{L, 1, \alpha}^{l}-\delta_{L, 4, \alpha}^{l}-2 \delta_{L, 7, \alpha}^{l} \frac{\tau_{3}^{3}}{2}-\frac{1}{2}-\delta_{L, 3, \alpha}^{l}\right. & \text { NU }\end{cases}  \tag{A.3}\\
& \Delta_{2,1, \alpha}^{l}= \begin{cases}1-\delta_{R, 1, \alpha}^{l}-\delta_{R, 4, \alpha}^{l} & \text { LR,HP } \\
\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l} & \text { LP,FP,UN } \\
\left(\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{2,2, \alpha}^{l}= \begin{cases}\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l} & \text { LR,HP } \\
0 & \text { LP,FP,UN } \\
0 \delta_{\alpha \alpha_{1}}+\left(\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}\right) \delta_{\alpha \alpha 2} & \text { NU }\end{cases} \\
& \Delta_{2,1, \alpha}^{l}= \begin{cases}1-\delta_{R, 1, \alpha}^{l}-\delta_{R, 4, \alpha}^{l} & \text { LR,HP } \\
\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l} & \text { LP,FP,UN } \\
\left.\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}\right) \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases} \\
& \Delta_{2,2, \alpha}^{l}= \begin{cases}\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l} & \text { LR,HP } \\
0 & \text { LP,FP,UN } \\
0 \delta_{\alpha \alpha \alpha_{1}}+\left(\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}\right) \delta_{\alpha \alpha_{2}} & \text { NU }\end{cases}
\end{align*}
$$

$$
\begin{aligned}
& \Delta_{2,1, \alpha}^{l 3}= \begin{cases}\left(1-\delta_{R, 1, \alpha}^{l}+\delta_{R, 4, \alpha}^{l}\right) \frac{\tau^{3}}{2}+\delta_{R, 3, \alpha}^{l}+\delta_{R, 7, \alpha}^{l} & \text { LR,HP } \\
\left(\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}\right) \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}^{l}+\delta_{L, 6, \alpha}^{l} & \text { LP,FP,UN } \\
\left(\left(\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l} \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}^{l}+\delta_{L, 6, \alpha}^{l} \delta_{\alpha \alpha_{1}}+0 \delta_{\alpha \alpha_{2}}\right.\right. & \text { NU }\end{cases} \\
& \Delta_{2,2, \alpha}^{l 3}= \begin{cases}\left(\delta_{L, 2, \alpha}^{l}+\delta_{L, 6, \alpha}^{l}\right) \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}^{l} & \text { LR,HP } \\
0 & \text { LP,FP,UN } \\
0 \delta_{\alpha \alpha_{1}}^{l}+\left(\left(\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l} \frac{\tau^{3}}{2}+\delta_{L, 5, \alpha}^{l}+\delta_{L, 6, \alpha}^{l}\right) \delta_{\alpha \alpha_{2}}^{l}\right. \text { NU }\end{cases} \\
& \Delta_{2, \alpha}^{l}= \begin{cases}\left(\delta_{R, 1, \alpha}^{l}-\delta_{L, 2, \alpha}^{l}-\delta_{R, 4, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}-2 \delta_{R, 7, \alpha}^{l} \frac{\tau_{3}}{2}-\frac{1}{2}-\delta_{R, 3, \alpha}^{l}-\delta_{L, 5, \alpha}^{l}\right. & \text { LR,HP } \\
\left(1-\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}-2 \delta_{R, 7, \alpha}^{l} \frac{\tau_{3}}{2}-\frac{1}{2}-\delta_{L, 5, \alpha}^{l}+\delta_{R, 7, \alpha}^{l}\right. & \text { LP,FP,UN } \\
\left(1-\delta_{L, 2, \alpha}^{l}-\delta_{L, 6, \alpha}^{l}-2 \delta_{R, 7, \alpha}^{l} \frac{\tau_{3}}{2}-\frac{1}{2}-\delta_{L, 5, \alpha}^{l}+\delta_{R, 7, \alpha}^{l}\right. & \text { NU }\end{cases}
\end{aligned}
$$

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